FFT ANALYSIS OF CHARGE DYNAMICS IN TUMBLING MILL

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CERTIFICATE

It is certified that the work contained in this thesis entitled "*FFT analysis of charge dynamics in tumbling mill* ", by *Sarbajit Dutta*, has been carried out under my supervision and that this work has not been submitted elsewhere for any degree

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December, 1998

Dedicated to ......

My Grandfather

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I would like to thank my seniors and friends in the hostel and my lab-mates for providing me a cordial, friendly environment during my stay in IITK.

Abstract

Charge motion analysis in a ball mill is of utmost practical importance. It is directly related to power draft, wear of the charge and liners, and breakage characteristics of the grinding material It is difficult and expensive to trace the frequent variations of the grinding parameters by traditional methods in commercial scale operation. Since mechanical grinding emits strong sound as well as vibration signals, the variations in these signals is expected to correlate with the profile of the charge.

In order to characterize the charge motion inside the mill, through vibration signals, accelerometers are used and the signals in time domain are transformed to frequency domain by a spectrum analyzer. The resulting spectrum is analyzed to establish the most predominant mode of charge motion.

The use of accelerometers and other such similar sensors to collect vibration data from the actual in the industry tends to be very expensive. Hence, for the purpose of this study, a general purpose simulator is used to generate vibration data pertaining to different sets of actual operating parameters. These spectra are analyzed to study the predominant mode of vibration of the charge In this study, two extreme cases were considered· mill with different lifter bars and mill without lifter bars. The effect of mill speed and friction in the charge profile are studied Through vibration signature analysis, a new non-invasive alternative is developed for monitoring the characteristics of the charge inside ball mills during operation.

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Chapter 1

Introduction

1.1 Statement of the problem

It is common knowledge in the field of mineral processing that mechanical grinding produces strong vibrations. The frequency pattern and the energies of the vibration signals contain information directly related to the operation state of the mill. In an industrial mill, the charge profile is of utmost practical importance. Fluctuations in the profile of the charge add to the variations in the power draft Ideally, the mill operator would like to run the mill at a constant level of power draft but owing to the fluctuations in the profile of the charge mill operation is typically done relying heavily on intuition and empiricism.

It is extremely difficult to visually monitor the profile of the charge of a mill under operation due to the harsh environment prevailing inside the mill. It also changes continuously due to changes in viscosity of the slurry, wear of liners and balls, change in ore characteristics, etc. It is easy to visualize that a square lifter wears out very fast, starting with the corners and spreading to the leading face. This causes a long term shift in the profile of the charge. At the same time, during any given revolution, the ball charge is also found to change from one mode of motion to another depending on the ore and slurry characteristics. These variations give rise to fluctuations and drop in the power draft.

Power draft is generally considered as an indicator of grinding efficiency. Therefore, it is essential to ensure that the mill runs at its optimum level of power draft. This optimal condition can be predicted by a good knowledge of the charge profile as a function of operating conditions of the mill. The accuracy of prediction of the charge profile comes through experience. Typically, the mill operator understands whether surging is occurring or not by listening to the mill noise. An operator with experience can distinguish the noise during surging from the other noise signals. To achieve good grinding result or even to come to a satisfactory decision, the operator's experience and long term association with the particular mill is of enormous importance.

Therefore, the problem is essentially that of predicting the charge profile for smooth operation of the mill. With the advent of the technology on signal acquisition, signal processing, system identification and advance computation, the mill operator's decision-making can be imitated by computerized instrumentation and numerical analyses. To this end, a general purpose simulator based on (discrete element method) can be utilized to generate the vibrational data in time domain. To evaluate the signals, the time domain waveforms can be transformed into frequency spectra by Fast Fourier transform (FFT) technique. The variations in the characteristics of the vibration signals can be correlated to the different charge profiles in the mill. The severity of a particular charge profile, distribution of impact energy, and the nature of impacting forces etc., can be accurately predicted using this general purpose simulator. Thus, this information if available is expected to improve the mill operation and set a new trend in plant practice.

1.2 Importance of motion analysis

Comminution is amongst the oldest technologies and despite tremendous research efforts, in a mineral processing, plant it still consumes a significant percentage of electrical energy. The ball mill, which is the work-horse of most comminution operations, is still very

inefficient, as much of energy is wasted in impacts which do not break particles, or is consumed in the generation of unwanted product sizes such as ultra-fines. In order to improve the efficiency of the ball mill by proper design and operation, it is essential to have a good understanding of the mechanics of the motion of the tumbling charge.

A systematic study of motion of the charge inside a ball mill was pioneered by Davis [1]. He gave an analytical solution of the trajectories of balls within the mill. Since then, many have analyzed the charge motion in ball mills to study the motion of a single ball as well as an en-masse motion of the charge. These studies have resulted in complete characterization of the motion of the charge into three distinct charge profiles, namely, cascading, cataracting and surging From operational standpoint, these charge profiles lead to variation in rate of grinding, energy consumption, throughput, wear of ball and liner etc. Motion analysis of the charge is therefore quite important from a practical standpoint. For example, accurate prediction of power draft cannot be done without a quantitative description of the profile of the charge [14]. However, a careful look at the literature reveals that this analysis is not interpreted adequately to address the more important question of mill efficiency.

The geometric profile of the charge corresponds to the most efficient milling can be determined in a qualitative sense by a thorough analysis of the motion of the charge under varying design and operational conditions. The practical importance of this information lies in the use of the knowledge of the individual ball trajectories in a mill to determine the speed at which the mill must be run such that descending balls fall on the toe of the charge, and not on the mill liner The impact of the balls on the liner plates can lead to unduly rapid wear adding to the operating costs. In addition, knowledge gained from the analysis of the motion of ball charge can be utilized for optimum design of the lifter bars.

1.3 Charge dynamics in a ball mill

There are generally four types of geometric profiles of the charge that can be identified in a ball mill under different design and operating conditions, namely surging, cataracting, cascading and centrifuging.

Surging

The phenomenon known as surging consists of a pendulum-like oscillation of a whole charge of the mill, within the mill shell. Thus, for one part of the cyclic motion, the charge is moving along the mill in the same direction as the mill shell, and during the second part of the cyclic motion, in the opposite direction. A schematic of this behavior is presented in Fig. 1 1 This oscillatory motion of the charge, leads to abrupt variations in the torque necessary to maintain the mill shell in steady motion. Sometimes, such demands upon the driving motor can trip the circuit breakers [2]. Excessive liner wear and cyclic mill noise are due to ball mass surging in the mill. So surging is indeed intrinsic to the ball charge. Due to friction between the outer layer of balls and the mill shells, a layer of ball moves up with the mill shell, but as the inclination of this mass increases, the downward force due to weight of the

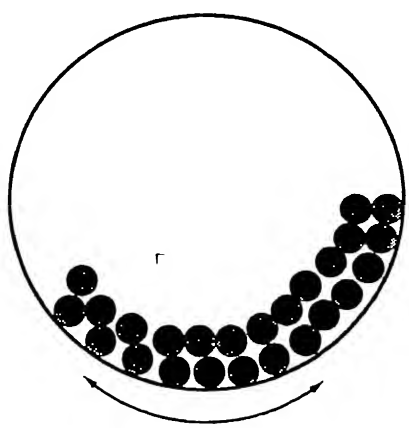


Figure 1.1: Schematic diagram showing surging profile

mass exceeds the upward frictional forces on the mill wall. And, hence the entire layer slides down and collapses. A recent investigation into this phenomenon was done by Vermeulen et al. [4] and Agrawala et al. [3] who have observed surging, due to fluctuations in the slip of grinding charge in rotary mills with smooth liners. These authors claim that surging is not an oscillating motion at moderate to high ball fillings, because the grinding media in the outer layers do not ever move counter to the mill shell, but found that at low ball fillings(J $\leq$ 30%), surging resembles the oscillating behavior of the charge as first reported by Rose and Sullivan [2].

The basic equation for surging in a mill according to Rose and Sullivan [2] is

$$\alpha = \phi(D/d, \mu, J) \tag{1.1}$$

where 2α is the angle through which the center of gravity of the mill charge oscillates, D is the mill diameter, d is the ball diameter, μ is the coefficient of friction, J is the mill filling and ϕ is some arbitrary function This equation is arrived at by dimensional analysis, therefore it cannot reveal the micro-mechanics of the surging process. However, it gives an idea about the effect of various design and operating parameters on surging.

Figure 1.2 shows that by monitoring conditions like mill diameter, ball diameter, mill filling and coefficient of friction it is possible to operate the mill in non-surging region. In a typical 13 - 16 ft diameter mill, the top size of the ball is 5 in. So the $\frac{D}{d}$ ratio is approximately around 40. If the filling of the mill is 30 percent, i.e., J= 0.3, then to ensure that the mill operates in the non-surging region the coefficient of friction must be greater than $\mu =$ "0.06/0.3" = 0.2 .

Cascading and Cataracting

In this motion, as shown in Fig. 1.3, the balls will travel on circular arcs, concentric with the shell of the mill, until the point of instability is reached, after which they roll down the surface, which is inclined at about 30 degree to the horizontal, in a series of parallel layers.

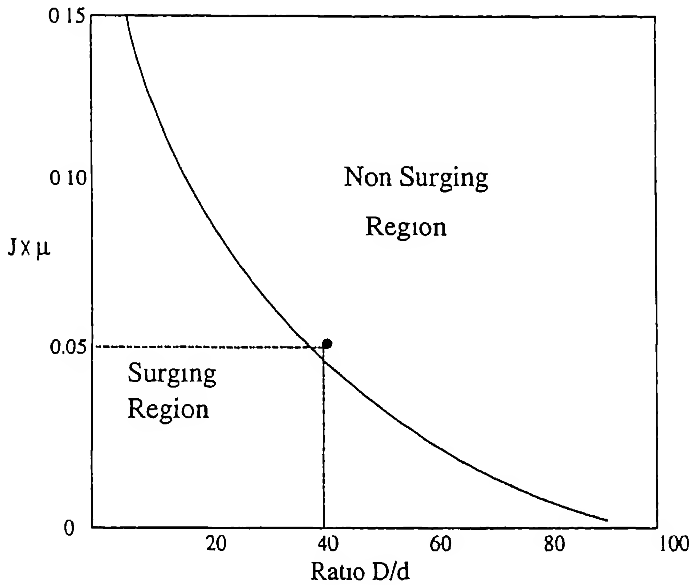


Figure 1.2: Effect of mill diameter, ball diameter, coefficient of friction and mill filling on surging.

This phenomenon is known as cascading.

At higher speeds of rotation, the balls no longer roll down the surface of the charge but at a certain point are projected into space and thereafter describe approximately parabolic

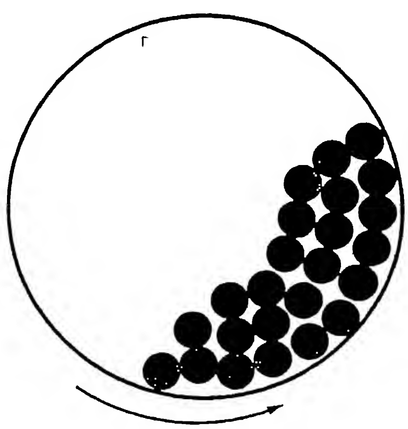


Figure 1 3· Schematic diagram showing cascading profile

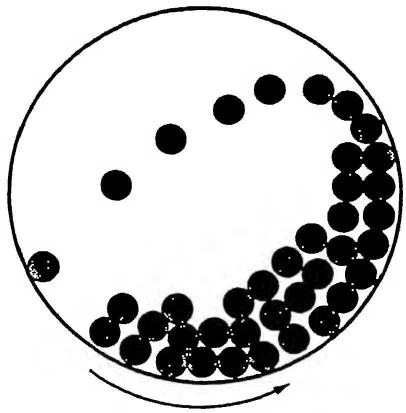


Figure 1.4: Schematic diagram showing cataracting profile

paths before again meeting the ball mass as shown in Fig. 1.4. This type of motion is known as cataracting.

During cataracting some balls hit the liners, hence a loss of energy is there without effective grinding. These type of collisions are not desirable. These high energy impacts are so intense that it is likely that some of them, either over grind the material, or simply get wasted in collisions with the mill liner [8]. The optimum energy distribution can be obtained in cascading motion relative to the other three motions namely cataracting, centrifuging and surging.

Centrifuging

When the speed of rotation of the mill is increased, the particles are projected with progressively greater velocities until the theoretical trajectory for a particle, lies lying against the mill shell. Clearly, since the particle cannot pass through the shell, it would lie against the shell throughout the cycle and so be carried around continuously with the mill shell as shown in the Fig. 1.5. This condition is known as centrifuging. The speed at which centrifuging just begins is referred to as the " critical speed ". It is easy to relate the critical

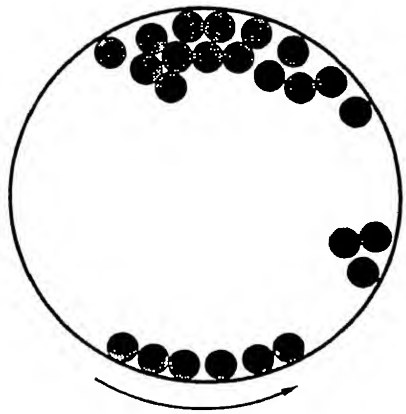


Figure 1.5: Schematic diagram showing centrifuging profile

speed, N, to the mill diameter, D in m, as

$$N = \frac{42.2}{\sqrt{(D)}} \tag{1.2}$$

In general, it has been found that the energy required to keep the charge in motion increases with mill speed up to the critical speed [9]. Beyond the critical speed when centrifuging begins, energy required for motion decreases with increase in speed as shown in Fig. 1.6.

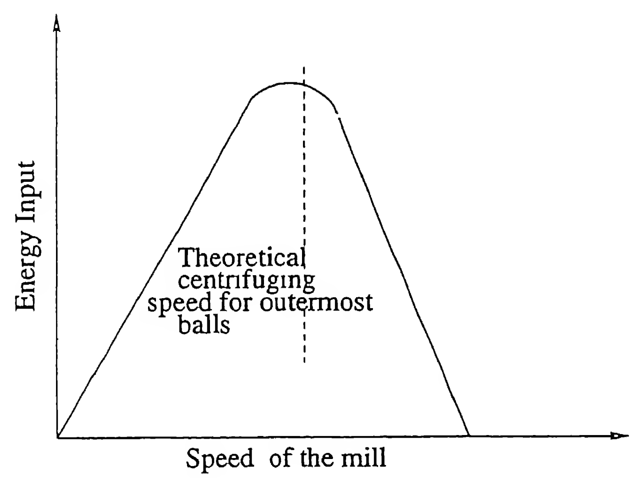


Figure 1.6: Relationship of energy input in ball mill to speed of mill [9]

All the four types of charge motion discussed above are closely related, i.e., a slight change in operating condition may change a particular motion to another type. During milling, it is almost impossible to monitor all the operational parameters. But, to have effective energy utilization, the mill must be operated in a particular mode which is often found to be the cascading mode. In the cascading motion of the charge, the number of impacts between the balls and between the balls and liners is optimum in comparison to the other three motion.

1.3.1 Experimental work relating to charge dynamics

Experimental measurements to determine the profile of the charge has been done in several ways Strain gauges, integrated circuitry in the ball, accelerometers, etc., have been commonly used to gather information that was correlated to the profile of the charge. Liddell and Moys [16] studied the effect of mill speed and filling on the position of the toe and shoulder of the ball charge and hence the torque. They used electrical conductivity probes inserted in the liner to locate the angular position of the toe and the shoulder.

Forssberg and Zeng [18] studied the vibrational characteristics of the mill by inserting an accelerometer to monitor the mechanical vibration and the acoustic pressure changes by a microphone. By principal component analysis and parameter identification, the variations of the grinding parameters were related to the changes of the source vibrational signals.

Rolf and Vongluekiet [6] have developed a new measuring technique to directly record the energy distribution inside the ball mill. This technique uses deflection of springs to measure force. When the impact force exceeds the pre-tension force of the spring, an electrical impulse is generated and sent to a counter built in the form of an integrated switching circuit inside the balls. These counts are read by means of a computer after the experiment. Rolf and Theudchai [7] used these instrumented balls to measure the energy distribution in the ball mill. This work re-iterated that the energy transfer is most effective when the charge motion

is predominantly of cascading type.

A vast amount of research work on motion analysis [3, 13, 14, 15, 16, 17] is also done by visual means in laboratory sized mills. In most instances these analyses resulted in mathematical models for the prediction of power draft. Obviously, in a real mill, motion analysis of the ball charge cannot be done by visual means. Indirect methods using sensors inside the prototype mills have found limited success due to prevailing harsh environment inside the mill Thus, information gathered by experimental means is still inadequate, primarily because the instrumentation itself is not well-developed Therefore, as of date, experiments with prototype mills have not been able to provide a complete description of the charge profile that can be used as a tool for proper mill operation.

1.4 Scope of Present Work

It is possible to analyze and correlate the vibrational signals coming from a mill shaft of a real mill to the prevailing known charge profile. However, even if the required instrumentation to record the vibrational data is available it is difficult to know the charge profile inside a prototype mill. Thus, it is envisaged that a simulator of a ball mill can be used to generate the vibration data as well as know the prevailing charge profiles. A general purpose simulator 2DMILL [11] is used for these purpose. Once implemented, this strategy shall lead to solving many operational problems of ball mills, and thereby improving the mill efficiency.

The purpose of this research is two-fold: first, to study the spectral characteristics of the vibrations of the mill shaft of a laboratory mill using accelerometers, and second, to identify characteristic frequency spectra associated with the four types of charge motion possible in a ball mill using the vibration data obtained from a general purpose simulator. The simulated acceleration time history of the ball charge is used to obtain the frequency spectrum of the charge mass by FFT analysis. After gaining confidence in correlating the frequency characteristics of the charge to the charge profile, it is possible to predict the

charge profile of a real mill under practical conditions.

Chapter 2

Frequency characteristics of charge motion

2.1 Fast Fourier Transform Technique

It is a common knowledge in the field of mineral processing that mechanical grinding as in the ball mill produces strong vibrations. The frequency content of the vibrations contains information directly related to the operating state of grinding [20].

The vibration signals are obtained by instrumentation and data acquisition. The time domain waveforms of vibration signals are recorded through accelerometers. In general, an acceleration signal wave can be represented in the form

$$a(t) = a_0 + a_1 \sin\omega_1 t + a_2 \sin\omega_2 t + \ .... \tag{2.1}$$

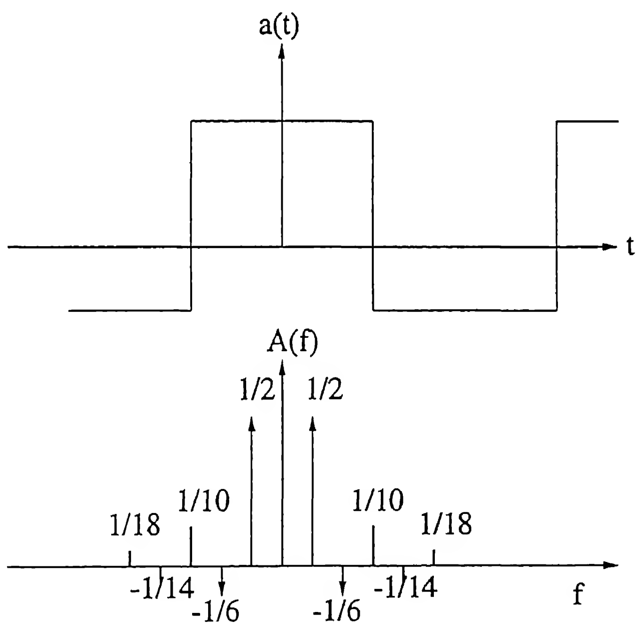
where a_0, a_1, a_2, ... are amplitudes corresponding to frequencies 0, ω_1, ω_2, ω_3,... The Fourier amplitude spectrum is a plot of the amplitudes "a_i" as a function of frequencies "ω_i". Mathematically, the Fourier amplitude A(f) corresponding to frequency "f" of the signal a(t) can be obtained as [22]

$$A(f) = \int_{-\infty}^{\infty} a(t) e^{-j2\pi ft} dt \tag{2.2}$$

where j is simply $\sqrt{-1}$.

A careful inspection of equation (2.2) reveals that if there are N data points of the acceleration time history and amplitude corresponding to N frequencies are to be determined, then the computation time is proportional to N^2, number of multiplications. To reduce this huge computation time Cooley and Tukey [5] developed the algorithm which is popularly known as the "Fast Fourier Transform" (FFT). This algorithm reduces the computation time of equation (2.2) to a time proportional to $Nlog_2N$ [21]. An example of the Fourier transform of a square wave function is illustrated in Fig.2 1. A square waveform can be decomposed into a set of sinusoids, the figure displays the amplitude and frequency of each sinusoid.

The original square wave, tells nothing of the sinusoids that it comprises of. A alternative way to display these various sinusoids is through the Fourier amplitude transform in



FOURIER TRANSFORM OF A SQUARE FUNCTION

Figure 2 1· Fourier transform of a square wave function

frequency domain which provide a graphic display of these harmonics.

2.2 Analysis of frequency spectra

In the Fig. 2 2, the different types of charge motion in a ball mill and the associated characteristic acceleration spectra are shown. These figures correspond to the ideal cases where only one single charge profile is taking place. But in real mills, this is generally not possible; a minute change in operating conditions may quickly change one charge profile to another

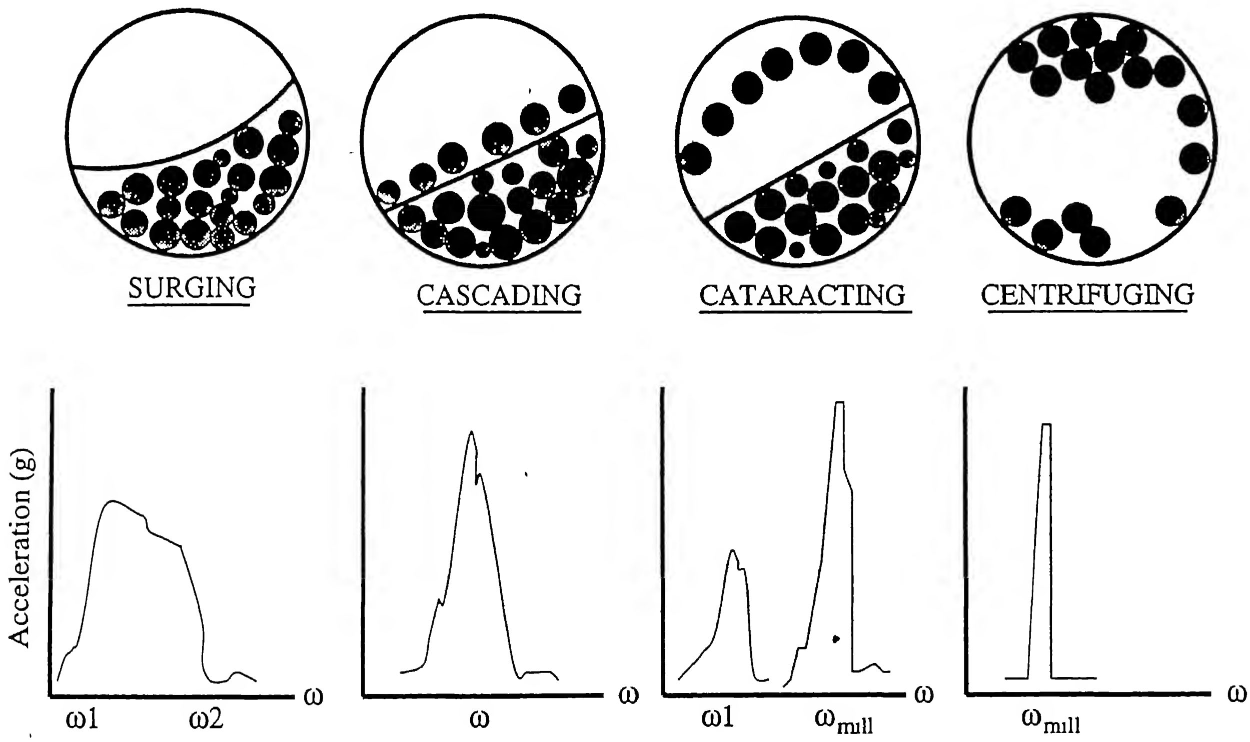


Figure 2.2 Various media motions and their corresponding acceleration spectra [23]

Surging

In surging due to oscillatory motion of the charge, the torque necessary to maintain the mill shell in steady motion fluctuates widely. The existence of such fluctuations may lead to collapse of the leading toe of the charge, when the charge, comes to rest at the end of the forward motion. This collapse imposes fluctuations of small amplitude but higher frequency upon the main motion; this, in turn, gives rise to a complex overall motion which involves a great number of harmonics, but of small magnitude, in the frequency spectrum [2] After the charge has gripped the shell, its ascent is expected at the same speed as the shell. Therefore, the frequency spectrum must indicate one peak at the mill frequency, and another at surging frequency. But the time durations for the ascent and the descent of the charge are not equal, and hence the peak accelerations during the ascent and descent of the charge are also not equal. Further, the descent of charge is a sudden phenomenon, similar to that of impact loading. So the frequency spectrum will reflect a band of frequencies over which the amplitude is significant. So there is no one unique characteristic frequency that corresponds to the surging frequency.

Cataracting

During cataracting motion, the balls rise along the mill wall up to certain height and then they are projectiled into space along approximately parabolic paths. Since the cataracting balls travel in a free space, their acceleration is very high. As the charge stick to the mill shell before this free fall, the frequency spectrum shows a peak corresponding to the mill frequency. Since, the cataracting is not along the mill shell, and since the charge is colliding with the mill shell with an impact at different locations, there is no unique characteristic frequency for cataracting. Several peaks can be seen in the spectrum depending upon the severity and number of impacts.

Cascading

During cascading, the ball charge tumbles down over its top surface, which is inclined to the horizontal at an angle (generally at 30 degrees) in a series of parallel layers. There is little variation in the acceleration during cascading due to more or less periodic charge motion. Therefore, there is no wide band in the frequency spectrum as in case of surging. Instead, there is a significant characteristic peak in the frequency spectrum, at the cascading frequency.

Centrifuging

At the critical speed the ball charge begins to centrifuge. When the charge centrifuges, the balls lie against the shell throughout the cycle of rotation. Thus, there is a single characteristic peak in the frequency spectrum at the mill frequency. But in a real mill, centrifuging rarely occurs. What is typically observed is that a portion of the ball stick to the mill shell when the slurry becomes too viscous. This condition may be reflected by a small-amplitude distinct peak in the frequency spectrum.

2.2.1 Single Ball Analysis

The correlation between various modes of charge motion and the frequency characteristics of the charge is illustrated by the detailed analysis of the trajectory of a single ball. A 12 in diameter ball mill is simulated with one ball. The acceleration time histories are transformed into the frequency domain through the FFT technique. The friction and speed of the mill are varied to achieve the different characteristic charge profiles and the associated frequency spectra.

Ball Trajectories under different conditions

Single ball simulation is done to identify the operating parameters such as friction, coefficient of restitution, and inertia of the ball. The variations in the charge profile due to changes in these parameters are studied. The single ball simulation gives only a rough idea of the charge profile, because in a real mill the environment is much more complicated owing to the presence of numerous balls.

Fig 2.3 shows these characteristic single ball trajectories obtained from the simulation. The charge profile corresponding to cataracting, surging and centrifuging are observed. Understandably, the cascading motion is not relevant in a single ball study. Fig. 2.3, shows that during surging, there is a oscillatory motion and the ball slips after it attains a particular height. During centrifuging, the path of the ball is that of the mill itself as the ball sticks to the mill shell. During cataracting, the ball is collides with the mill shell at different positions.

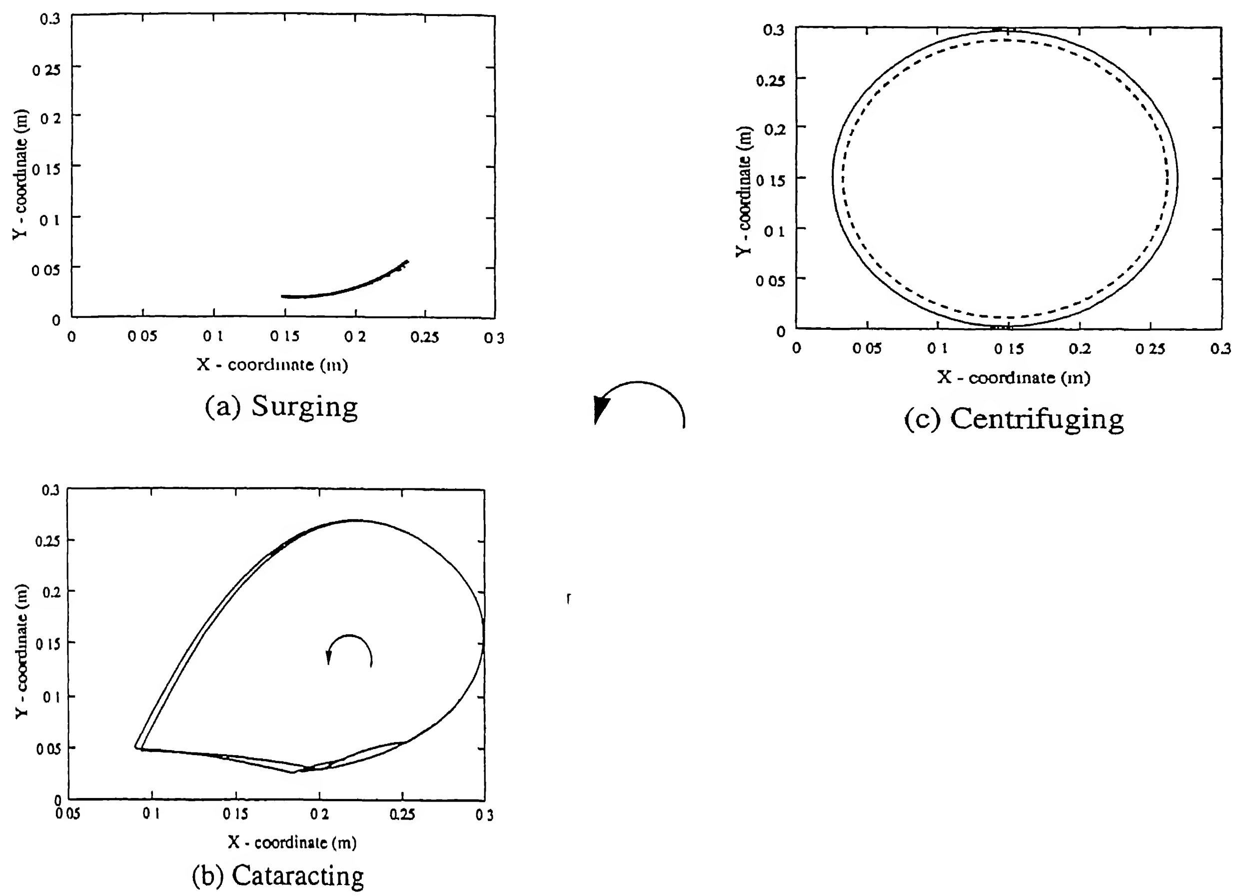


Figure 2 3: Single ball trajectories in a mill during surging, cataracting, and centrifuging.

FFT with single ball

The acceleration of the ball in the x-direction are shown in Fig. 2.4 for the three cases of charge motion, namely surging, cataracting and the centrifuging. In case of surging, three significant peaks are worth considering. The first major peak at 1.15 Hz corresponds to the mill speed, i e , 85 % of the critical speed of the mill of 8.06 rad/sec. As the ascent of the charge in assumed to be equal to that of mill frequency, this peak also corresponds to the ascent of the charge during surging motion. The second significant peak is due to the descent of the charge along the mill, as the frequency during the descent will be higher due to sudden collapse of the charge. Also, within a band of frequency, the amplitude is significant and this is a unique characteristic of surging profile.

In case of centrifuging, as long as the ball is in contact with the mill shell, there will be one major peak at 1.542 Hz corresponding to the mill speed i.e., 120 % of the critical speed of 8 06 rad/sec.

In case of cataracting, the ball initially glides along the mill and eventually gets projectiled into a parabolic path After it leaves the mill shell, it falls on to the mill shell in free flight. Thus, there must be one characteristic frequency corresponding to the mill speed, and the other frequencies correspond to the speed at which the ball impinges on the mill shell. But in the spectrum shown in Fig.2.5(b), the first frequency at 1.4 Hz is not exactly equal to the mill speed, because it is found that the gliding speed of the ball along the mill shell is slightly higher than the speed the mill. Several other peaks arising in the spectrum correspond to the impacts between the ball and the mill shell. This is purely a problem of contact mechanics used to compute the speed of the ball. Nevertheless for a mill speed of 1.28 Hz, one peak corresponding to the ball speed is observed at 1.4 Hz and the other peaks due to impact of the ball on the mill shell depending upon the severity of impact.

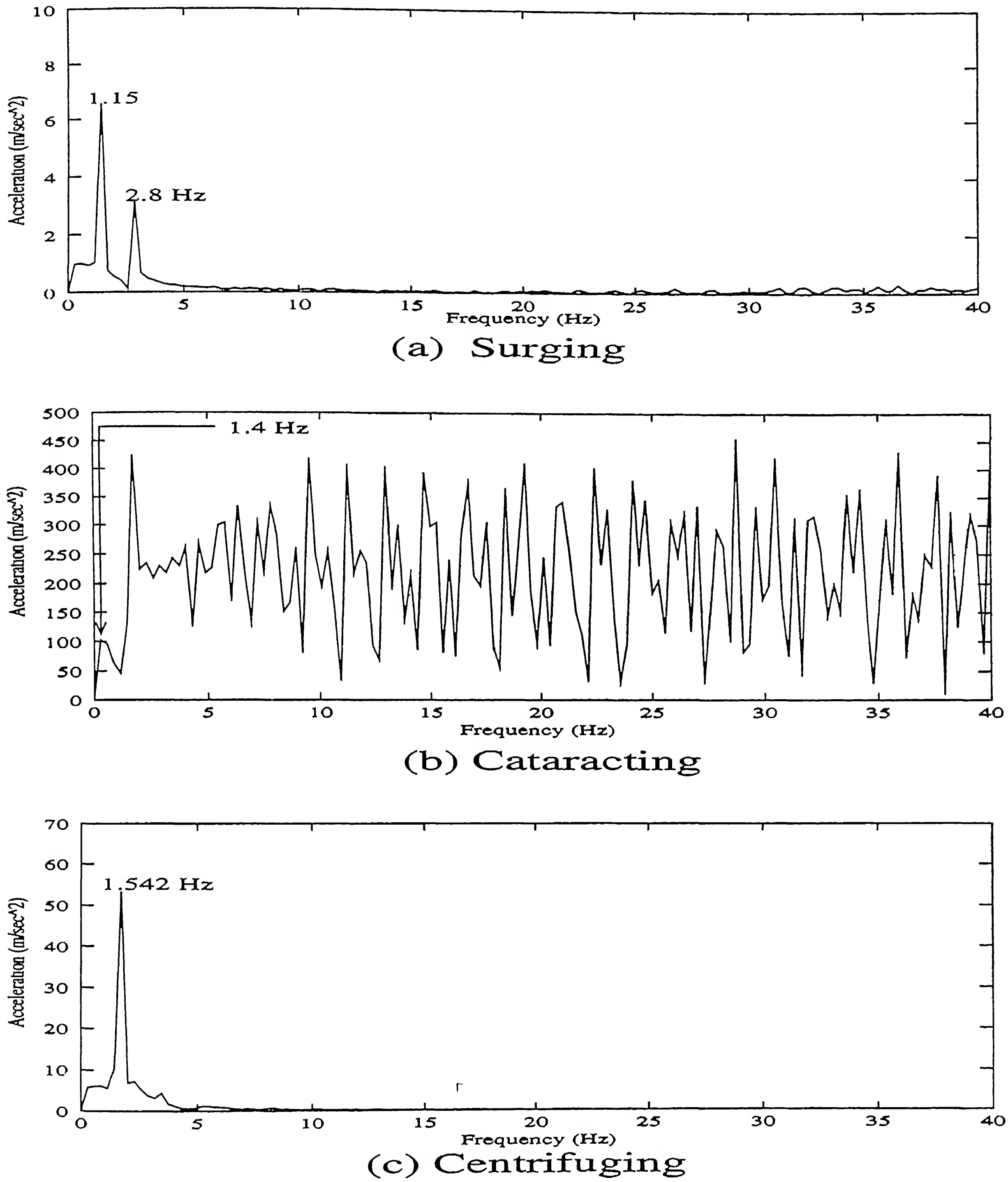


Figure 2.4: Acceleration spectra in y direction in case of surging, centrifuging and cataracting with single ball

In an industrial mill, due to the presence of large number of balls and due to the complicated environment, the charge profile is complex. It is almost impossible to achieve a steady profile of the charge in cataracting conditions because the operating parameters change continuously. Even though, the frequency spectrum is much more complicated in a real mill, the major characteristic of the spectrum corresponding to a particular charge profile is expected to be similar to that of the single ball. Thus, this feature can be taken advantage of to characterize the charge profile in laboratory mills as well as industrial mills.

Chapter 3

Frequency analysis of charge motion in a laboratory mill

This chapter deals with the experimental work performed to study the motion of the charge in a laboratory scale ball mill. A general purpose simulator based on the discrete element method is used to numerically simulate the motion of these balls. The frequency spectra obtained from the numerical simulation are validated against the experimental data. This forms the basis for using the simulator as a diagnostic tool for predicting the charge profile of industrial ball mills discussed in chapter 4.

3.1 Experimental set up

The grinding system consists of a laboratory size ball mill of diameter 12 in and length 11 in. The critical speed of this mill is 8.06 rad per second. The mill is deliberately made without liners so that the surging behavior of the charge can be studied. Figure 3.1 shows the layout of the grinding system. This figure also shows the instrumentation used for signal acquisition and processing. The charge consists of steel balls of density 7.86 t/m^3.

The instrumentation consisted of two accelerometers and a spectrum analyzer. The accelerometers are (FBA - 11's manufactured by m/s Kinematrics Inc, USA) of ±1g. One

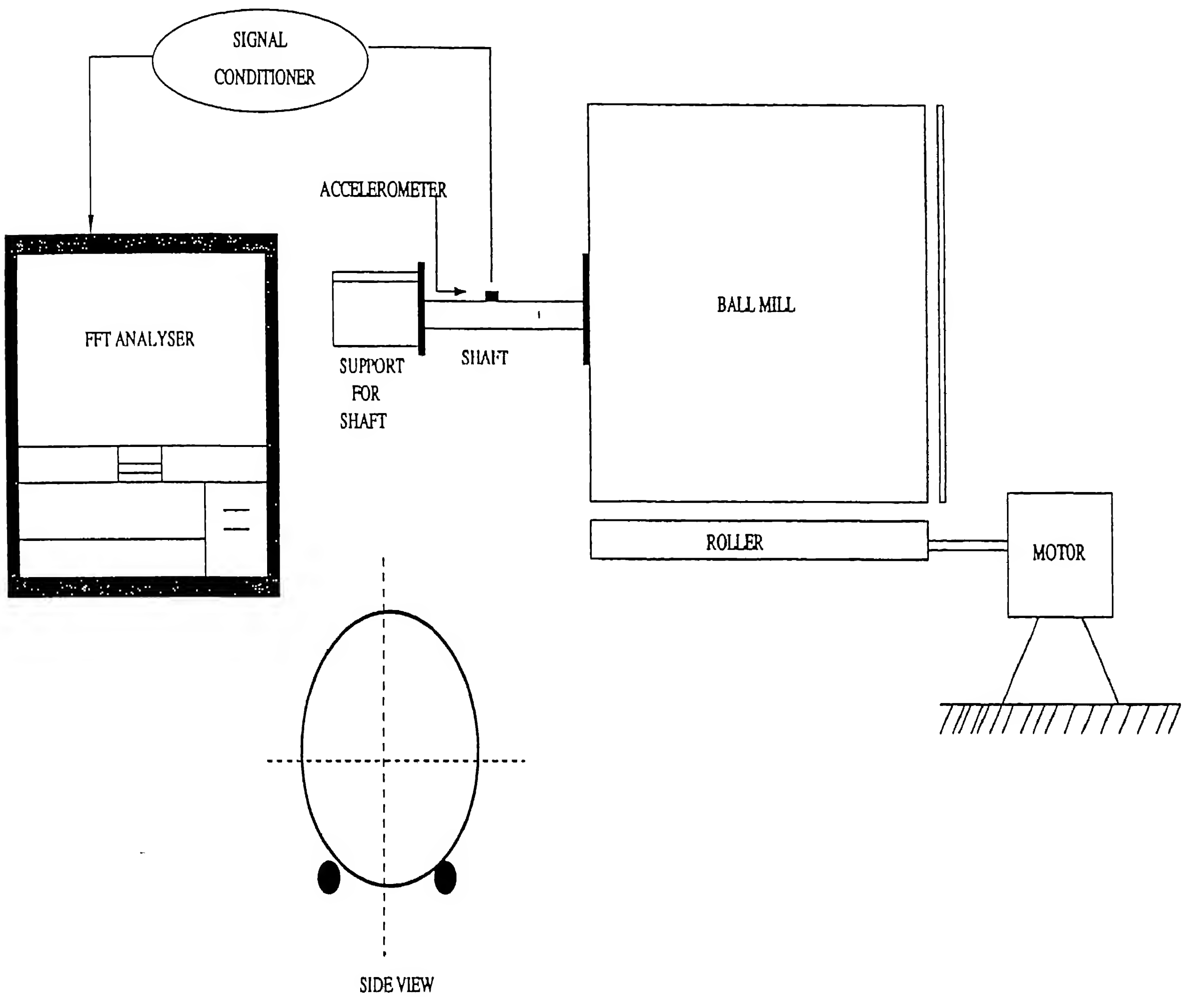


Figure 3.1: Experimental set-up showing a ball mill; accelerometers are mounted on the shaft to record the vibrations.

accelerometer is positioned along the shaft to measure vibration along the horizontal direction and the other along the vertical direction as shown in Fig. 3.1. The acceleration signals obtained through the signal conditioner (VSS -II manufactured by m/s Kinemetrics Inc, USA) and fed to a spectrum analyzer (HP3852A manufactured by m/s Hewlett Packard Inc, USA) which transforms the time domain data to frequency domain spectra by the FFT technique.

Table 3.1: Details of the mill and the charge used in the experimental work.

Mill Type	Ball Mill
Mill diameter	12 inch
Mill length	11 inch
Mill speed	44,63,75 and 88 rpm
Ball Load	25,30 and 37 Kg
Top size of the ball	1 inch
Ball material	Steel

Different motion characteristics were captured by changing the mill speed. Also the mass of the charge in the mill is varied as 25 Kg, 30 Kg and 37 kg. The ball mill was run at speeds of 44, 63, 75, and 88 rpm. Once the mill response is considered to have become steady, the acceleration time histories of the system from the shaft are sampled by the spectrum analyzer and an averaged frequency spectra of the acceleration signals are obtained. Details of the mill and the charge used in this study are given in Table 3.1 .

3.2 Experimental Results

3.2.1 Ambient Vibration

Even without operating the mill, the low level ambient vibration of the mill can be captured by sensitive accelerometers. Figure. 3.2 shows the ambient vibration spectrum obtained from the laboratory mill with 25 Kg charge mass. This spectrum reflects the vibration level of the surroundings of about 0.04 g and its frequency content along with the natural frequencies of the system.

3.2.2 Effect of Mill Speed

The mill is operated at different speeds with a charge mass of 25 Kg. Fig. 3.3 to 3.8 show the frequency spectra of the vertical acceleration time history of the shaft.

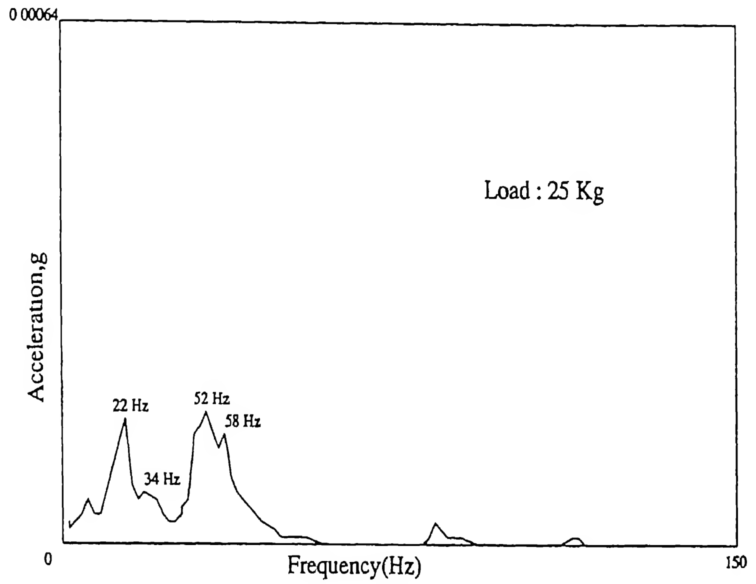


Figure 3.2: Frequency spectra of vertical acceleration at ambient condition.

Spectrum at 44 rpm

Figure 3.3 shows the vertical spectrum of acceleration at 44 rpm. It shows that the charge profile is predominantly of surging type. In the figure there are large number of frequencies within a broad band of spectrum, i.e., between 20 Hz and 110 Hz. In surging, due to the oscillatory motion of the charge many frequencies are present within a broad spectrum of frequencies having comparable amplitudes. The peak frequency at 112 Hz is due to cantilever oscillation of the mill The mill is revolving with the roller. If proper alignment is not there between the mill shell and the roller then during rotation of the mill this will induce cantilever oscillating mode frequency in the characteristic spectrum. This is a typical dynamics problem

Vibration at 63 rpm

Figure 3.5 shows the acceleration spectrum at 63 r.p.m. It is found from the spectrum that the charge profile involves both cascading and cataracting. The frequencies between 12

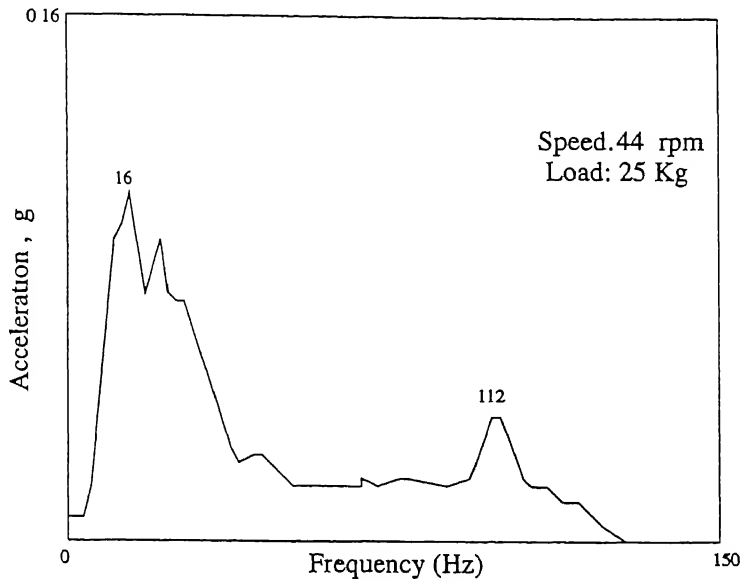


Figure 3 3· Frequency spectra of vertical acceleration at 44 rpm

and 40 Hz are due to the cataracting of the charge. At this speed the outer layer of the balls which are in contact with the mill shell are only expected to cataract. The remaining portion of the charge cascade. Ideally, the motion expected is shown in the figure 3.4. Only the outer layer of balls are cataracting and the major portion of the charge is moving in a cascading fashion. A unique value of frequency at 112 Hz corresponds to the cantilever oscillating mode frequency During cascading as minimum number of impacts is taking place the amplitude of the cascading frequency is not high as shown in the figure. The broad spectrum with many peak frequencies between 40 and 105 Hz represents the surging profile of the charge.

Vibration at 75 rpm

The speed of the mill is increased to 75 rpm to study the effects of cataracting in the mill. The acceleration spectrum obtained in the spectrum analyzer by FFT technique is shown in Fig. 3.6. The frequency band between 16 and 50 Hz have the typical twin peak characteristics of cataracting profile. The two isolated peaks in the frequency spectrum

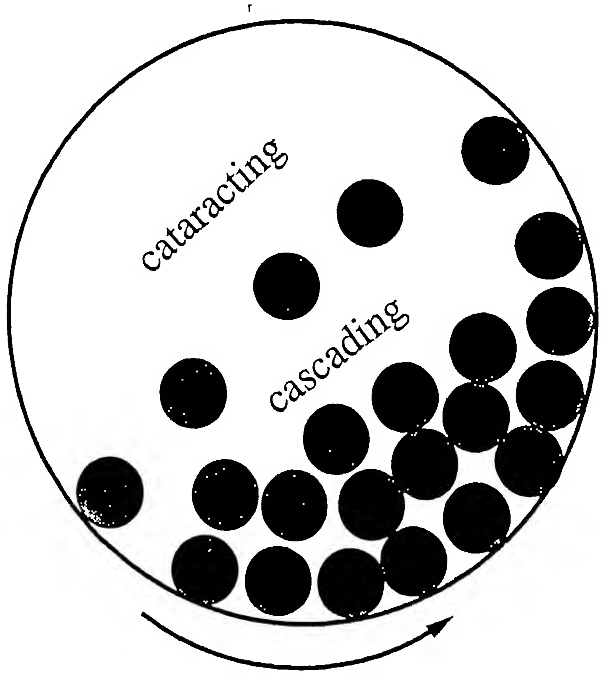


Figure 3.4: Expected Motion at 60 % of critical speed

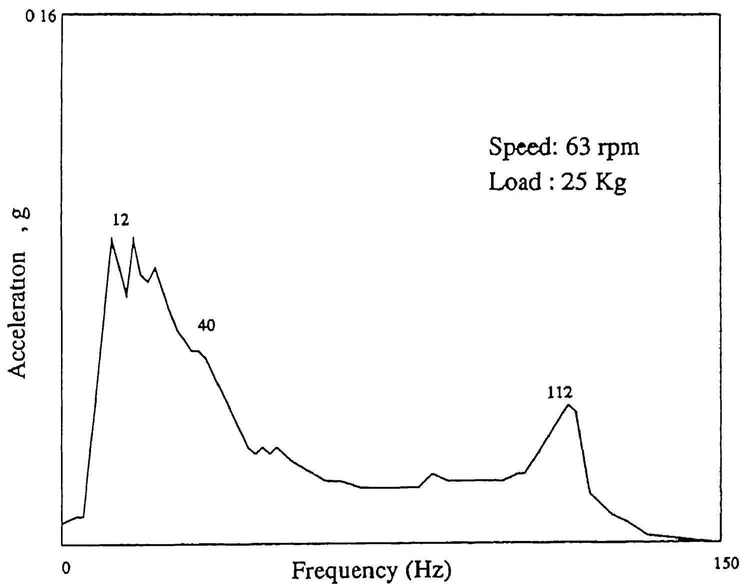


Figure 3.5: Frequency spectra of vertical acceleration at 63 rpm

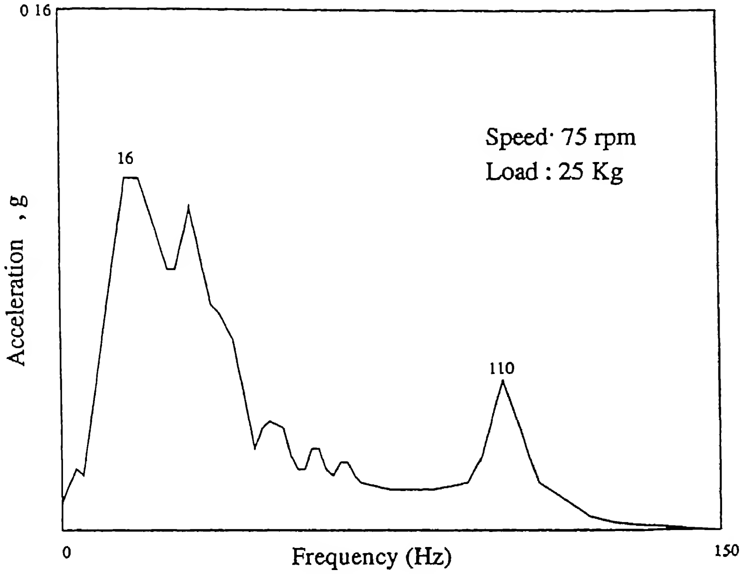


Figure 3.6: Frequency spectra of vertical acceleration at 75 rpm

represents cataracting behavior of the balls. The frequencies from 55 to 95 Hz have typical wide band characteristics of surging. The unique characteristic peak at 110 Hz corresponds the cantilever oscillating mode frequency.

Vibration at 88 rpm

The speed of the mill is further increased to 88 rpm to study the effects of cataracting with increase in mill speed. It is found that the frequencies between 44 to 90 Hz have characteristics of cataracting process as shown in Fig. 3 7. More than two peaks are seen implying that the cataracting capture is not steady, but occurring at different frequencies.

3.2.3 Effect of mill filling

To characterize the effect of mill filling, different charge mass was tried. Not much change in the spectrum is observed under these different loading conditions. The acceleration spectrum remained almost unchanged but only the magnitude of acceleration varied under

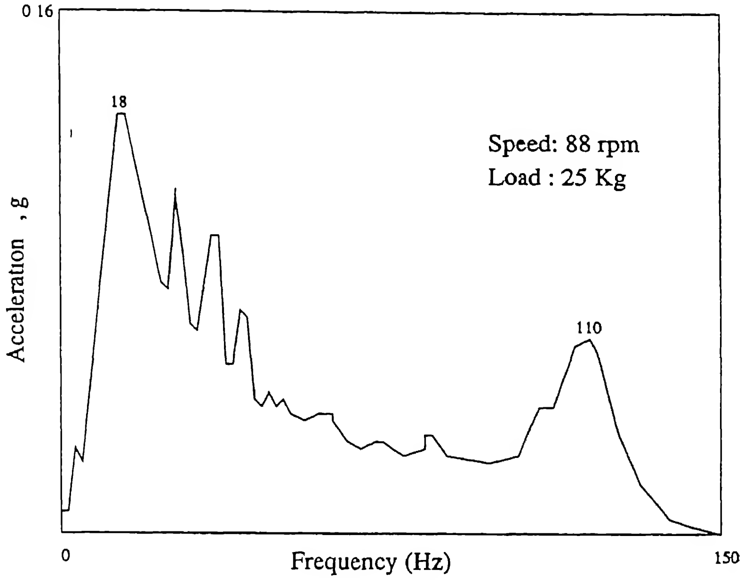


Figure 3.7: Frequency spectra of vertical acceleration at 88 rpm

various loading conditions. Frequency domain spectrum obtained with ball load of 30 Kg is shown in Fig. 3.8. The spectrum shows that the charge is predominantly cataracting. The frequency band between 20 to 40 Hz represents cataracting motion of the charge. The broad band of frequencies with low amplitude from 58 to 80 Hz is due to oscillatory motion of the charge, i.e., surging. Comparing 75 rpm spectrum for two different loads, namely 25 Kg and 30 kg, shows that the nature of the spectrum is almost same. Difference is only in the amplitudes of the characteristic peaks. In the spectrum with load 25 Kg the peak frequency at 110 Hz corresponds to the cantilever oscillating mode frequency, while in the spectrum with 30 Kg load under this frequency comes down to 96 Hz This is expected since a larger mass would reduce the frequency in proportion to the square root of the mass, i.e., $110\sqrt{25/30}$ 100 Hz, which is close to 96 Hz.

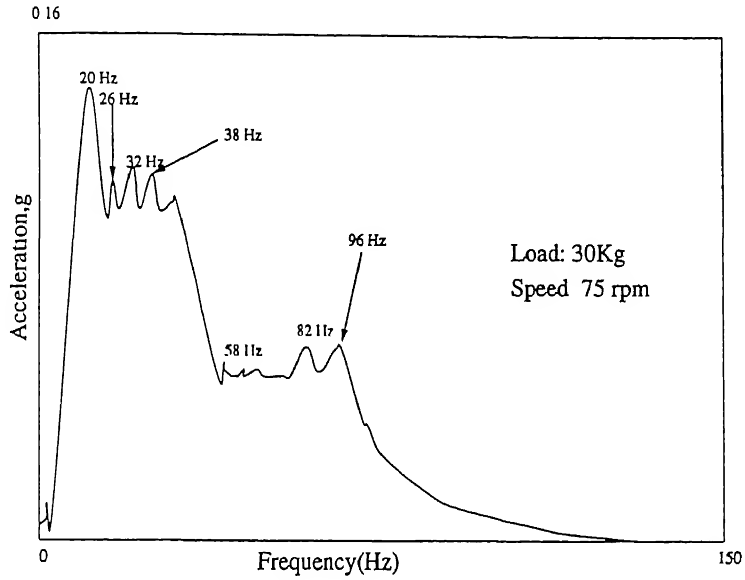


Figure 3.8: Frequency spectra of vertical acceleration at 75 rpm (load 30 Kg)

3.3 Numerical Simulation

A ball mill simulator based on the discrete element method is used to analyze the motion of the charge. The simulator requires the following input: mill speed, ball load, mill dimension, and other material parameters such as stiffness, coefficient of restitution and friction. The simulation provides the position, velocity, and acceleration of each and individual ball comprising the charge mass

At the end of every simulation, the acceleration time history of each ball is used in the following manner

- The accelerations in the two orthogonal direction of all balls are summed up at the start of each time step.
- The average acceleration time history is computed by dividing the total acceleration time history with number of balls.

- The frequency spectra of these average accelerations time histories are obtained by the FFT technique.

In the simulation the mill is considered as individual slices in which the ball-charge motion is confined to the radial direction only After simulating one such slice, the results of the entire mill are calculated from the known number of slices comprising the real mill. This total number of slices is the actual ball weight divided by the weight of the discs in a single slice. The mill is treated as a series of two dimensional compartments, each containing one layer of balls. Naturally, the compartmental approach simplified the real three dimensional problem to two dimensions.

The simplified flow chart of the FFT technique is shown in Fig. 3.3 .

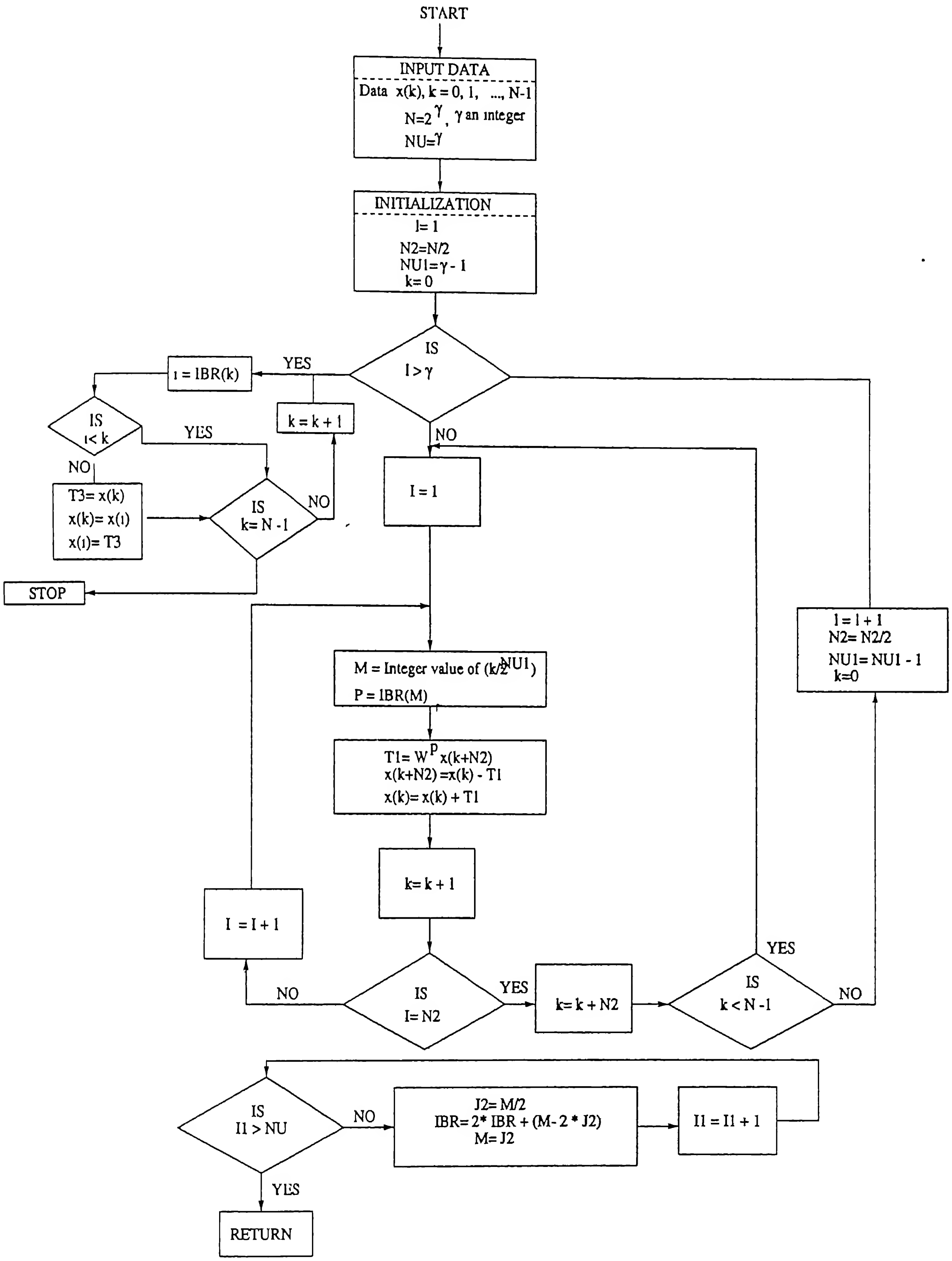


Figure 3.9: Flow chart used in this study to compute FFT of a signal

3.4 Simulation Results

3.4.1 FFT with 40% of mill filling, and coefficient of friction 0.6

The motion of 18 balls of 1 in diameter inside a 12 in diameter mill is simulated. This corresponds to 40% mill filling which is close to the case of laboratory mill considered in this study. The acceleration of all the balls is summed at the end of each time step and the resulting time history in vertical direction is converted to the frequency spectrum by FFT technique. Several simulations were carried out by varying the coefficient of friction and the speed of the mill to arrive at the different profiles. The coefficient of restitution is kept constant at 0.45 for ball-wall contact, which is the typical value as found in the literature [11]. The critical speed of the mill was 8.06 rad/sec and the simulations were carried out at 60, 70, 80, and 90 % of the critical speed. The other data used in the simulation are given in Table 3 2.

FFT at 60 rpm

The mill is operated at 60% of the critical speed and the vertical acceleration spectrum from the simulation is presented in Fig. 3 10. This figure shows peaks which are harmonics of one another. The initial peak is at 7.36 Hz and the others are at simple harmonics of it. Here, within a band there is are many frequencies which is expected due to the oscillatory

Table 3.2: Simulation data for a 12 in diameter mill.

Normal Stiffness	400000 N/m
Shear Stiffness	300000 N/m
Coefficient of friction	0.6
Coefficient of restitution: ball-ball impact ball-wall impact	 0.6 0.45

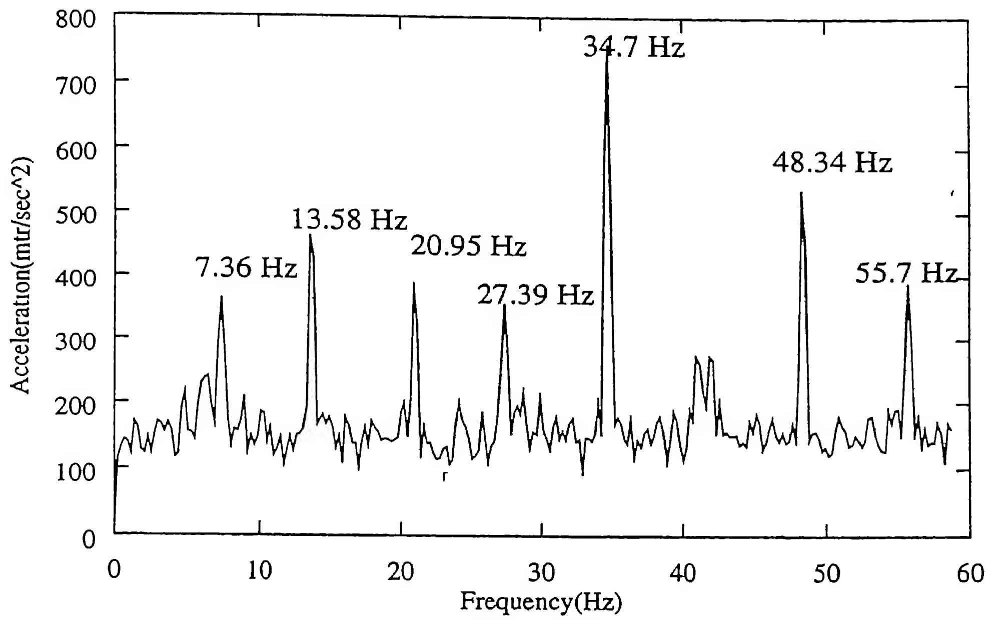


Figure 3 10 Frequency spectra of vertical acceleration at 60 % of the critical speed

behavior of the charge inside the mill during surging. The peak frequency in every band is due to the sudden collapse during charge descent. Depending upon the severity of these collapse, the amplitudes are changing accordingly.

FFT at 70 and 80 rpm

Figures 3.11 and 3.12 show the the vertical acceleration spectra corresponding to 70 and 80% of critical speed; At these speeds the charge motion is a mixture of cascading and cataracting. In Fig 3 11 corresponding to 70% of critical speed, it is observed from the frequency peaks that the motion is predominantly cascading type. During cascading motion the charge is maintained in specific manner within the mill. Due to tumbling action of the balls within a short period of time these high peak frequencies between 38.9 Hz to 41.4 Hz are arising. The other peaks are due to the cataracting of the balls, which are impinging either on the mill shell or the toe of the charge. In Fig.3.12 the peak frequencies between

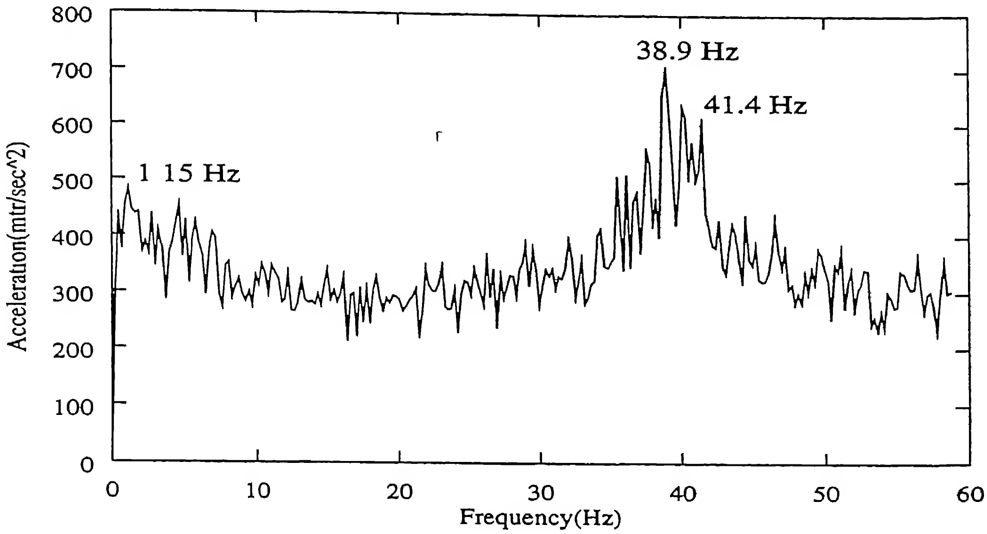


Figure 3.11: Frequency spectra of vertical acceleration at 70 % of the critical speed

44.2 Hz and 45 81 Hz are due to tumbling action of the ball mass with one another. In other words the peaks are due to the cascading profile of the charge. Other isolated peaks are due the cataracting of the balls.

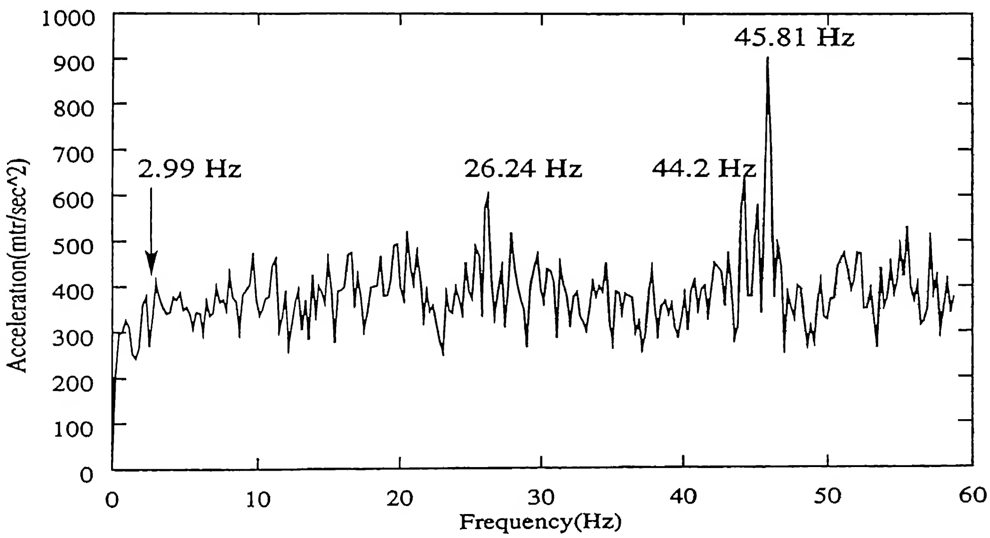


Figure 3.12: Frequency spectra of vertical acceleration at 80 % of the critical speed

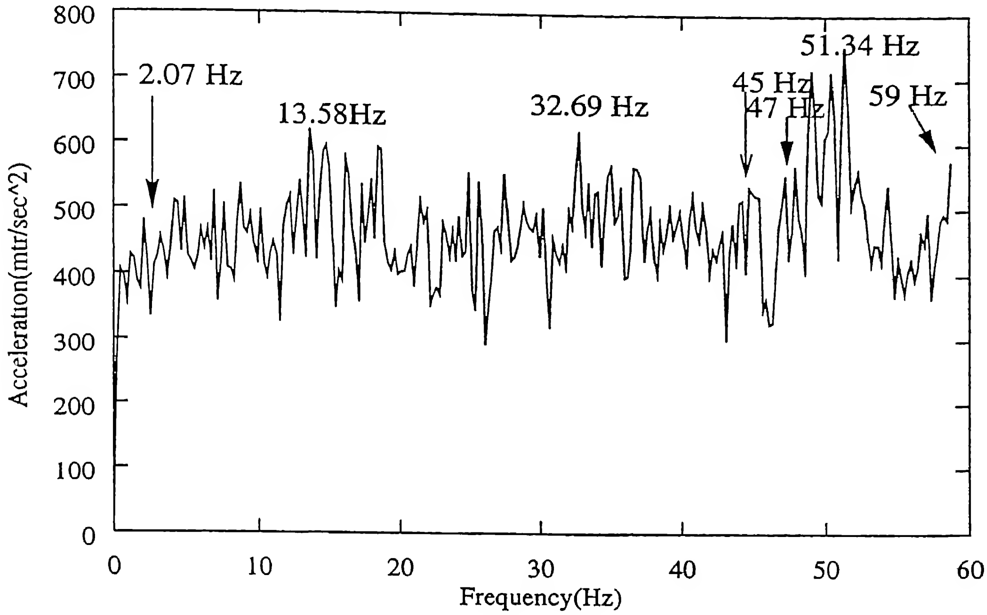


Figure 3.13: Frequency spectra of vertical acceleration at 90 % of the critical speed

FFT at 90 rpm

At 90% of the critical speed, the frequency spectrum shown in Figure 3.13 indicates that the motion is mainly of cataracting type. This is due to the fact that there are several isolated peaks which are not harmonic to each other. These peaks are due to the charge making impact with the mill shell. Centrifuging is not occurring as the mill is a smooth one.

Figures 3.14 - 3.17 shows surging, cascading, and cataracting when the summation of the force in the vertical direction on the mill is considered. The same trend as in the case of acceleration spectra is observed. But in case of centrifuging, there is not only one peak at 1.5 Hz which is the mill speed in Hz (i.e., 120% of the critical speed), but there is another significant peak due to a fraction of the charge that is cataracting. In Fig. 3.14 the peak frequencies at 26 Hz, 45 Hz etc are due to the sudden collapse of the charge mass in surging motion. In Fig. 3.15 the peak frequencies between 37 Hz and 39.9 Hz are due to the tumbling action of the ball mass The isolated peaks within the entire spectrum in Fig. 3.16 is assumed

to be due to cataracting of the charge mass.

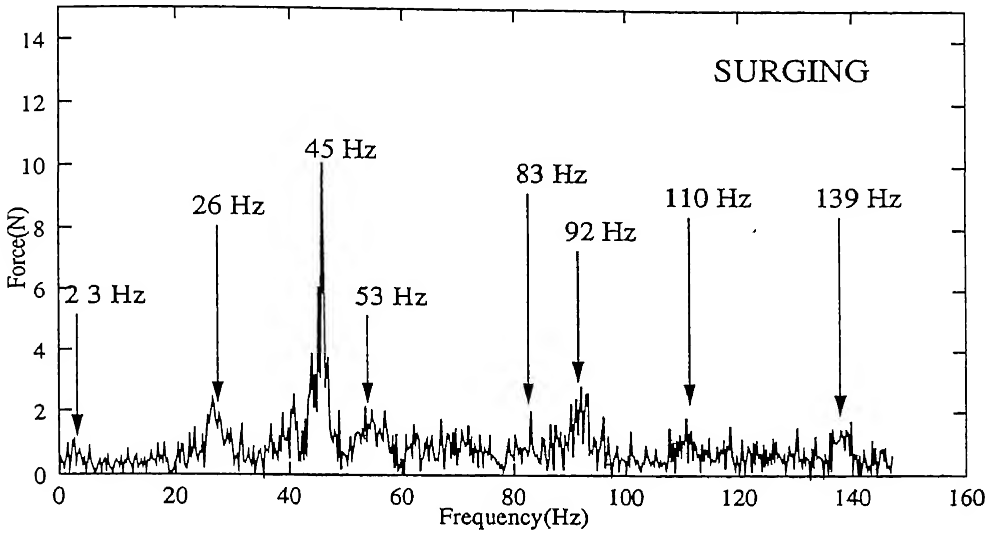


Figure 3.14. Frequency spectra of vertical force in case of surging of the charge

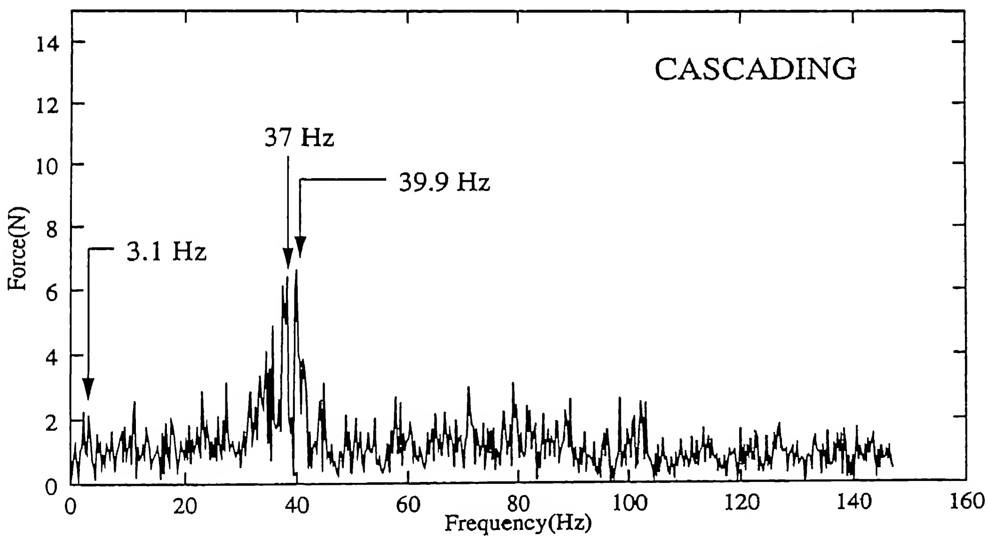


Figure 3.15: Frequency spectra of vertical force in case of cascading of the charge

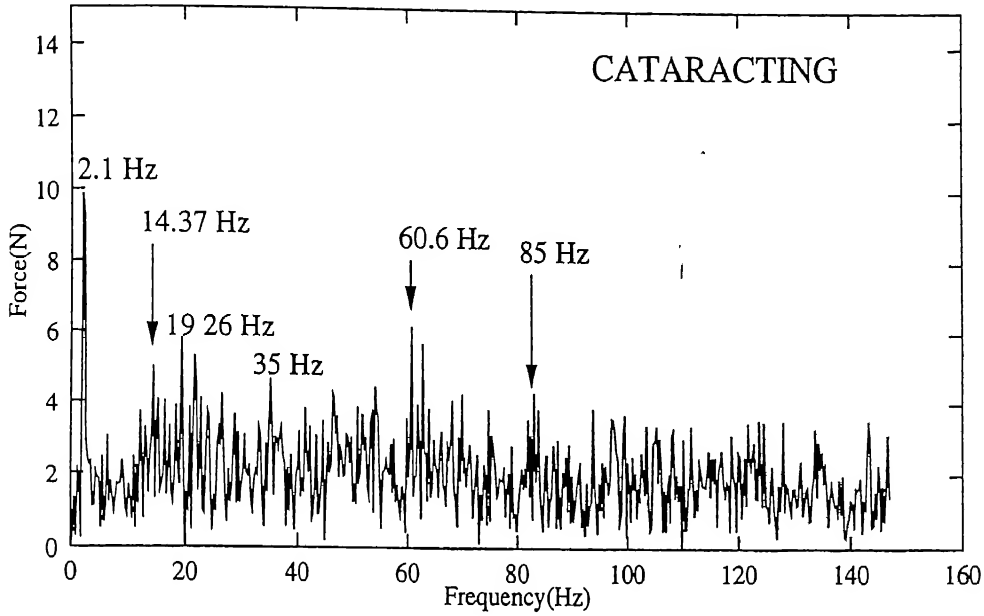


Figure 3.16: Frequency spectra of vertical force in case of cataracting of the charge

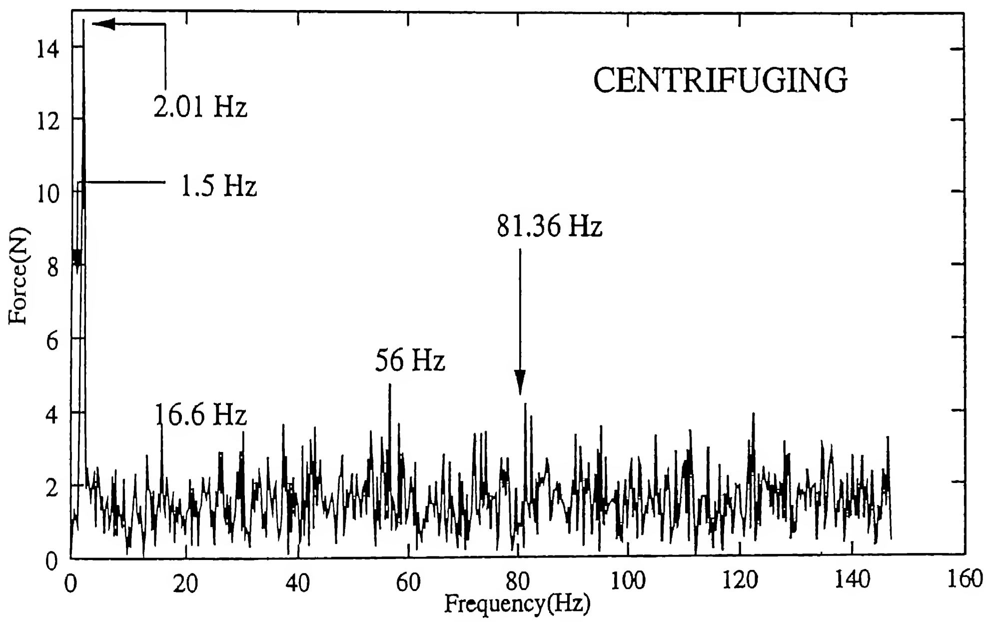


Figure 3.17: Frequency spectra of vertical force in case of centrifuging of the charge

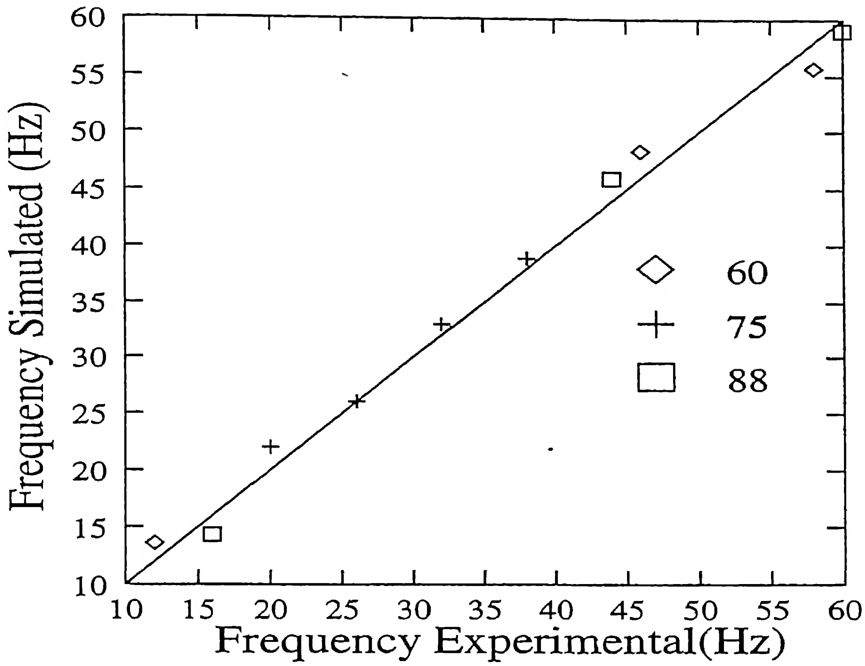


Figure 3 18: Comparison between the mill frequency obtained by simulation and experiment

3.5 Comparison of experimental and simulation results

To implement the general purpose simulator as a diagnostic tool in monitoring the operating conditions in a large mill, the simulation and the experimental frequencies are compared in the Fig. 3.18. The major frequencies are compared in both the experimental and simulation under similar conditions. It is clear from the figure that the frequencies are quite comparable.

3.6 Discussion

Under similar operating conditions, the peak frequencies of both the experiment and simulation are quite comparable. Even though exact correspondence is not there. This may be because of the following reasons:

- In the simulation by the general purpose simulator few material properties like coefficient of friction and restitution, stiffness, damping constant are input parameters. The exact values of these simulation parameters within a mill are difficult to asses. The simulations are carried out using the values proposed in the literature. This difference in simulation parameters may have contributed to some variation in the spectrum.

- The sensor location and orientation have strong effect in picking the correct signals from the mill. The vibration spectrum differs with the sensor orientation by referring to the rotation axis of the ball mill. After carrying out careful experiments successfully in an large scale ball mill it was proposed by Forssberg et al. [19] that the lower frequencies of spectrum are independent of sensor location. The higher frequencies are dependent on both location and orientation of the sensor.

- In the experimental work the sensors are located in the shaft to measure the acceleration signals of the system. On the other hand, the simulator uses contact mechanics problem to generate the acceleration data. For this reason some difference in the magnitude of the acceleration data can be there

- In the simulation the mill is treated as a series of two dimensional compartments, each containing one layer of balls. The three dimensional collision problem is simplified to two dimensions. Due to this simplification some variation may be there between the spectra of a 3D laboratory mill and 2D simulated mill.

Chapter 4

Industrial Application of Vibration Analysis

This chapter deals with application of quantitative analysis of charge motion to tackle some of the operational problems associated with mill operation under industrial environment. Here the frequency domain spectrum of the acceleration of the ball charge of typical industrial mills (4.87-m diameter) is considered. Simulations of industrial mill both with and without liners are carried out under various operating conditions. This is done to identify the effect of key operating variables on the profile of the charge.

4.1 Milling Practice

Grinding mills used in the industry are typically 13 to 18 ft in diameter. These mills run at 60 to 85% of critical speed and up to 40% of mill filling on a continuous basis. The ball charge is supplied to the mill intermittently. Typically 3 to 4 different sizes of balls are fed to the mill. Over prolonged usage, these balls wear out. Thus at any given time there exists an equilibrium distribution of balls from the top size to the smallest size comparable to size of the grate. Similarly, the lifter bars also wear out during operation. In addition, ore characteristics change from shift to shift causing changes in the behavior of the

slurry which eventually lead to over-load condition. There are other problems that affect the smooth functioning of the mill. malfunctioning of hydrocyclone in close circuit with the mill, variation in breakage rates of particles, water addition rates, etc.

The above mentioned problems typically lead to variations in the profile of the charge which in turn affect the power draft. Thus, it is conceivable that if the charge profile is known then it is a matter of judgment to predict the parameter that is affecting it. It can be done in a precise manner with the help a mill simulator such as 2DMILL [12]. The methodology adopted here is to run the simulator based on existing mill data to build a data base for motion analysis. The motion can be viewed by a real time animation program. The vibration data is analyzed to generate a frequency spectrum of the charge motion. It is clear that these two sets of information supplement each other. For example, in case of surging, motion analysis of the charge by computer animation may not reveal the intensity of impacts and the frequency of surging cycle. At the same time, frequency spectrum may not show the finer details of ball to ball interaction of the overall motion. For example, cataracting of the charge can be easily identified through the frequency spectrum, but it may be difficult to predict whether or not the cataracting balls are hitting the liner (mill shell) causing damage.

4.2 FFT of liner-less mill

The motion of the charge in a large mill is no different from what is observed in laboratory mills. In other words, motion characteristics such as surging, cascading, and cataracting are common to both types of mills. However, pure surging of the charge can be easily identified in a liner-less mill, although this type of milling practice is quite rare.

A 4.87 m (16 ft) diameter mill without lifter bars is considered for simulation. The critical speed of the mill is calculated to be 2.0025 rad/sec, i.e , 0.23 Hz. The mill is simulated with 800 balls with varying diameter and total ball load of 1533.8 Kg. This is just a two dimensional section of the complex three dimensional ball charge as the motion of the charge

Table 4.1. Simulation data for a 4.87 m dia mill without liners

Mill Type	Ball Mill without Liners
Mill speed	90 % of the critical speed
Ball Load	1533.8 Kg
Ball material	Steel
Normal Stiffness	400000 N/m
Shear Stiffness	300000 N/m
Coefficient of friction	0.1, 0 3, 0.6 and 0.9
Coefficient of restitution:	
ball-ball impact ball-wall impact	0.6 0.45

in the transverse direction is of primary interest. These are time intensive computations that also require lot of disk space for data storage. Thus the analysis is only restricted to two revolution. Furthermore, the numerical data pertaining to second revolution alone is considered for spectrum analysis in order to discard the initial inertia of the charge. The simulation parameters are given in detail in Table 4.1.

All the simulations are carried out to study the effect of friction on the surging behavior of the charge. Surging is known to be strongly dependent on the coefficient of friction [2] that can be directly related to the characteristics of the slurry.

Effect of friction

In order to identify and correlate the characteristic of the charge under the influence of friction, several simulations were carried out by changing the coefficient of friction while keeping the mill speed constant. The frequency spectrum corresponding to the motion of the charge at a coefficient of friction of 0.1 is presented in Fig. 4.1. The peak at 0.23 Hz corresponds to the mill speed. The wide band of frequencies between 1.0 Hz and 2.5 Hz are the characteristic of surging profile, as explained in earlier chapters.

The coefficient of friction of the mill was increased from 0.1 to 0.9 incrementally. The

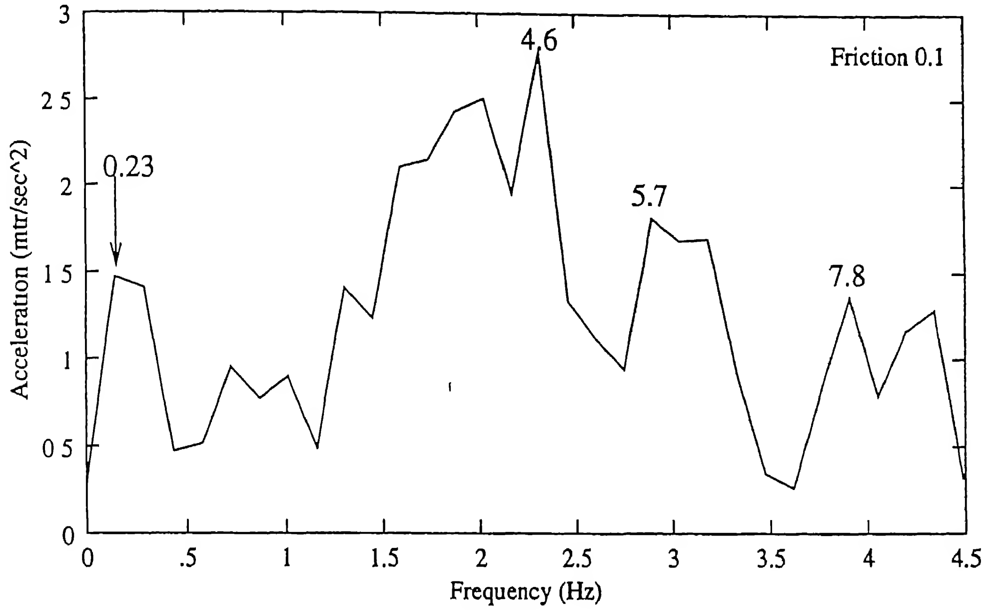


Figure 4.1: Frequency spectra of vertical acceleration of large mill without liners (Friction 0.1)

frequency spectrum corresponding to these simulations is shown in Figure 4.2 to 4.4. It is clear from these figures that as the coefficient of friction increases the tendency of the charge to surge decreases. Furthermore, with increase in the coefficient of friction, number of peaks corresponding to cataracting balls also increase. Figure 4.4 which corresponds to the charge motion at a coefficient of friction of 0.9 shows maximum number of cataracting peaks. Nevertheless, even at a coefficient friction of 0.9 there is still a fraction of the charge that is found to have a surging tendency. The frequency range between 2.3 and 3.2 Hz shows many frequencies within a band of spectrum. This is again a typical characteristic of surging due to cyclic motion of the charge. The other peak frequencies are due to the cataracting balls colliding with the mill shell. In all the figures the characteristic peak corresponding to the mill speed is present at 0.23 Hz.

The spectrum analysis in case of a liner-less mill suggests that coefficient friction plays

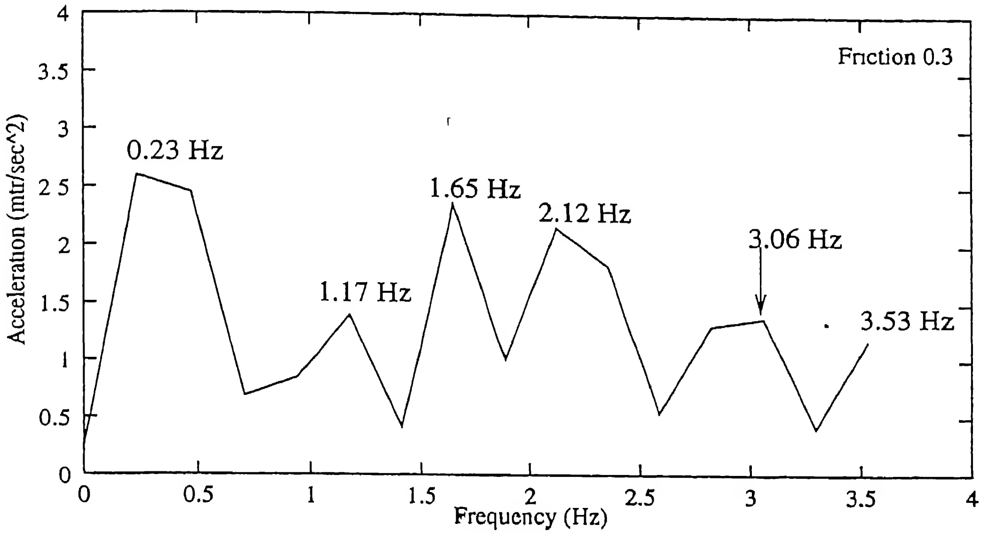


Figure 4.2: Frequency spectra of vertical acceleration of large mill without liners (Friction 0.3)

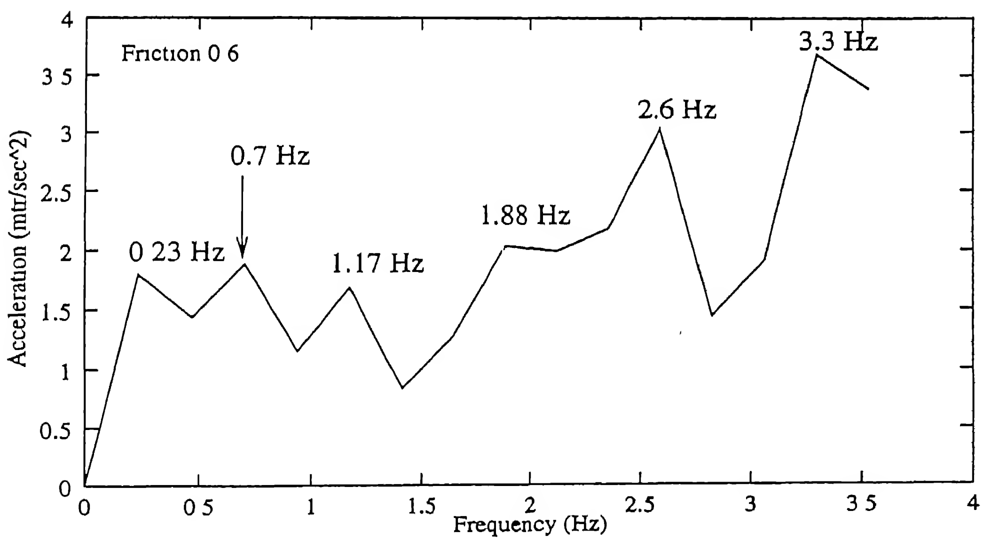


Figure 4 3: Frequency spectra of vertical acceleration of large mill without liners (Friction 0.6)

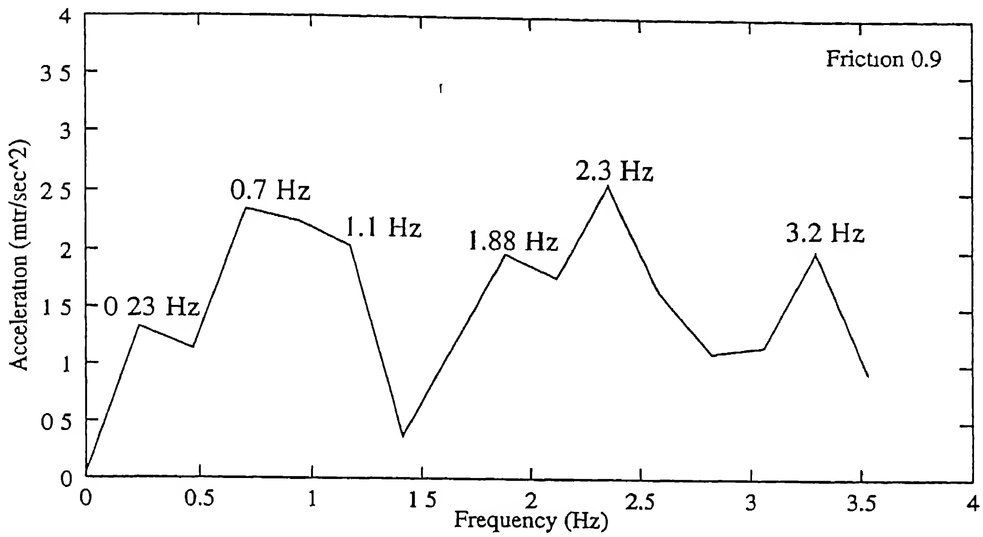


Figure 4.4: Frequency spectra of vertical acceleration of large mill without liners (Friction 0.9)

an important role on the overall profile of the charge. It is seen that with the increase in coefficient of friction the tendency of surging decreases while that of cataracting increases. Surging, at higher values of coefficient of friction is believed to be taking place between layers of balls. However, when the coefficient of friction is low then the entire ball mass may surge. Between these two extremes, the amount of balls surging and the severity of their impact may vary.

4.3 FFT of large mills with liners

The industrial mill is always fitted with lifter bars as part of the liner. For a given rotating speed and filling of the mill, the impact velocities of the balls and the nature of the forces developed inside the mill can be controlled by the most favorable configuration of the lifter bars. Cataracting and surging is the major problem associated with the industrial mills The wear rate, impact energy distribution [8], number of wasted impacts without doing

efficient grinding are directly related to the number of wasteful impacts during cataracting motion of the charge.

It is important to study the motion of en masse grinding media for different profiles of lifter bars. After weeks of milling, alterations in the lifter bar shape cause changes in the ball trajectories, which affect the power draw. A 4.87 m (16 ft) diameter mill with two different types of lifter bars namely rectangular and wavy is simulated. 30 lifter bars are used in both the simulation. The schematic diagram of the lifter bars is shown in Fig. 4.5. With prolonged use, the rectangular lifter lends to wear and eventually act as wavy lifters which with finally wears to totally to become a smooth mill.

The mill is simulated with 832 balls and total ball load of 1569.7 Kg. The simulation is done for two revolutions only. The numerical data obtained from the second revolution is only considered for spectrum analysis in order to discard the initial inertia of the charge.

All the simulations are carried out at constant coefficient of friction 0.6 and 85% of the critical speed. The simulation parameters are same as presented in Table 4.1.

The animation snap shots capturing typical motion characteristics are shown in Fig. 4.6 It is clear from Fig. 4.6 that the ball charge under the action of a rectangular lifter moves in both cascading and cataracting manner. But the cataracting balls are impinging on the mill liner but in case of wavy lifter-bars the cataracting balls are falling on the toe of the charge instead of mill liner In the smooth mill the entire charge surges due to the low coefficient of friction between the outer layer of balls and the mill wall.

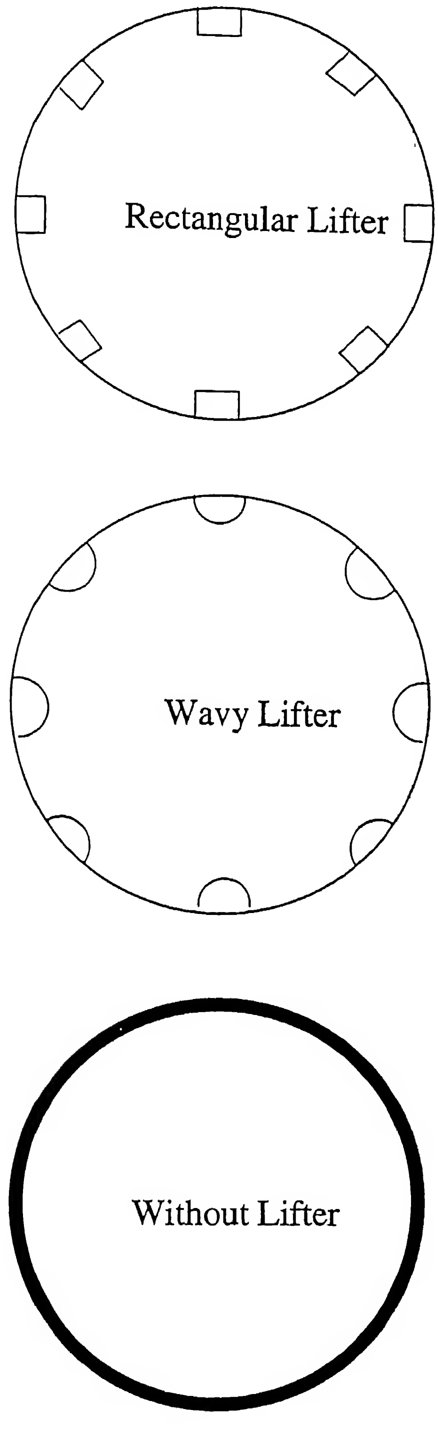


Figure 4.5. Different lifter bar profiles

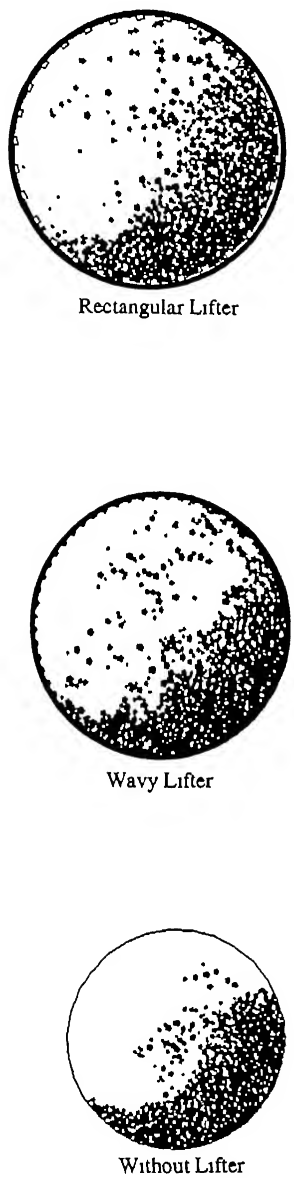


Figure 4.6: Animation snapshots of typical charge profile in different lifter-bar configurations

4.3.1 Large mill with different lifter configurations

The acceleration spectra with rectangular lifter shows isolated peaks which are characteristic of cataracting profile. The peaks are arising due to the impacts between the ball and the liner. In case of wavy lifter bar the magnitude of these peaks are lower, which confirms that the balls are not hitting the liners rather they are falling on the toe of the charge. The broad spectrum between 7.5 and 15 Hz confirms the fact that surging is taking place in case of the smooth mill. Also the amplitude of the vertical acceleration is much lower as expected in surging

4.4 Discussion

Prediction of charge profile inside a mill is of tremendous practical importance. The wear of the liners, power draft, breakage rate of the material are directly related to the profile of the charge. Charge profile can be predicted by the use of sensors. The sensors are very sensitive to its surroundings, in an industry, it is unrealistic to measure the vibration signals accurately Moreover, these sensors are often quite sophisticated and expensive. In absence of these sensors, use of a general purpose simulator is proposed. It is used to predict the charge profiles in an industrial mill. Due to huge requirement of data storage facility, only two extreme cases were considered. mill with liners and without liners. The motion of the charge from their characteristic spectrum is analyzed. The results of simulations that it is possible to predict the charge profile in its entirety utilizing this general purpose simulator. This simulation technique provides a new alternative for developing a monitoring system for grinding, which is expected to accurately predict the charge profile with continuously changing operating parameters.

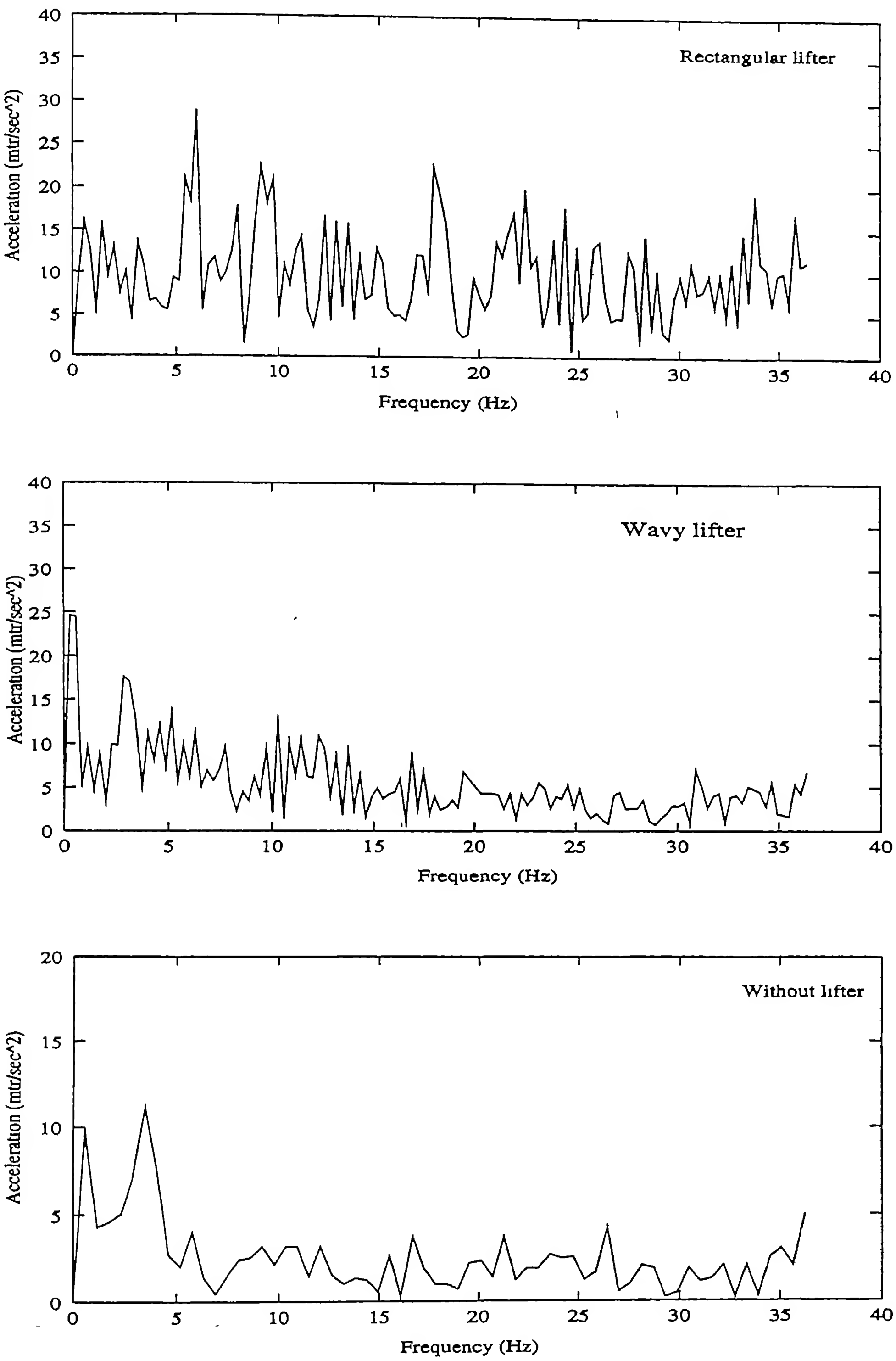


Figure 4 7: Frequency spectra of vertical acceleration of large mill with different lifter-bar configuration

Chapter 5

Conclusions

In ball milling a good knowledge of charge profile is very important. The power draft, breakage characteristics of grinding material, constant wear of liners and the grinding charge, impact energy distribution, etc., are directly related to the charge profile of the mill. Charge profile has been predicted in various ways, but its success is limited only to laboratory size mills. There is no sound technique available that can be implemented in the industry.

In order to predict the charge profile, vibration signals from a laboratory mill were recorded using accelerometers. The time domain signals were transformed to the frequency domain by a spectrum analyzer which basically does a fast Fourier transform on the time history data. Several experiments are carried out using different mill speeds to characterize the effect of speed on the profile of the charge. The results show that at the lowest speed (44 rpm) there is always a tendency of surging of the charge between a layer of balls and the mill shell. This is ideally the case when the mill smooth, i e., without lifters. At 44 rpm and 63 rpm, characteristic frequency of cascading profile is clearly identified in their corresponding spectrums. It is also observed that at these speeds only the outer layer of balls which are in contact with the mill shell have the tendency to surge or cataract; other balls move in a cascading manner.

It is known that with the increase in the speed of the mill, the severity of cataracting

increases. This is quite evident when all the frequency spectrums corresponding to 44, 63, 75, and 88 rpm are analyzed.

Experiments were carried out with various loading conditions to characterize the effect of filing on the charge profile. Not much change in the frequency spectrum is observed from the frequency spectrums except the magnitude of the peak frequencies increased.

A general purpose simulator 2DMILL [12], is utilized to generate the acceleration signal in time domain. These signals are then transformed in frequency domain with the help of fast Fourier transform technique. To start with, single ball trajectories were considered to identify and correlate the the characteristic features of the spectrum to various charge profiles. Then motion of 18 balls were used to compare the charge motion with that of the experiments. It is found that the peak frequencies under similar operating conditions are quite comparable. Several other simulations were carried out by changing the speed of the mill to observe its effect on the charge profile. It was observed that at a lower speed, i.e., 60 % of the critical speed, the ball charge is predominantly surging. With the increase in speed of the mill, more and more balls took cataracting trajectories and made direct contact with the mill shell giving rise to isolated peaks typical of cataracting profile.

A novel scheme is adopted to predict the charge profile of large mills under different operating conditions. Several simulations were performed using two types of mills: mill with different liners and without liners. The frequency spectrums of the mill without liners show that there is always some degree of surging irrespective of mill speed. But with the increase in speed the ball charge tend to shift from surging to cataracting. It is found as evident from the literature that surging is strongly dependent on the friction which is affected mainly by the characteristics of the slurry The mill with liners shows that surging can be minimized by the use of lifter bars. The cataracting balls which are impacting with the mill liner directly in case of rectangular lifter-bars can be optimized using wavy lifter-bars. With the increase in speed increased number of balls tend to cataract. It is evident that the

operating parameters of the mill can be monitored for the desired charge profile by analyzing variations in the vibration signatures of the mill. This is going to provide useful information for effective operation of the mill.

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